

The Standard Backward/Forward Sweep Power Flow

Paulo M. De Oliveira-De Jesus, *Member, IEEE*

Abstract

This paper describes a convenient formulation of the Backward/Forward (BW/FW) Sweep Power Flow applied to radial distribution systems with distributed generation. Proposed formulation was applied in a illustrative test system.

Index Terms

Backward/forward sweep, load flow, power flow, distribution system analysis

I. INTRODUCTION

SEVERAL Backward/Forward (BW/FW) sweep algorithms have been discussed in literature. In 1967, Berg presented a paper which can be considered as the source for the all variants of BW/FW sweep methods [1]. Later, a similar approach was presented in [2] based on ladder network theory. The BW/FW Sweep algorithms use the Kirchhoff laws. Different formulations can be found [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [15], [14]. BW/FW sweep methods typically present a slow convergence rate but computationally efficient at each iteration. Using these methods, power flow solution for a distribution network can be obtained without solving any set of simultaneous equations. In this work, the standard BW/FW sweep power flow is reformulated in convenient form. An illustrative four-bus example is solved.

II. STANDARD BACKWARD/FORWARD SWEEP POWER FLOW METHOD

The input data of this algorithm is given by node-branch oriented data used by most utilities. Basic data required is: active and reactive powers, nomenclature for sending and receiving nodes, and positive sequence impedance model for all branches.

In the following, the standard BW/FW sweep power flow method is written in matricial notation using complex variables. Branch impedances are stated as a vector \mathbf{Z} corresponding to a distribution line model containing a series positive sequence impedance for line or transformer. Shunt impedances are not considered in this first approach. Fig. 1 shows a radial distribution network with $n + 1$ nodes, and n branches and a single voltage source at the root node 0. Branches are organized according to an appropriate numbering scheme (list), which details are provided in [3].

$$\mathbf{Z} = \left[\bar{Z}_{01} \quad \dots \quad \bar{Z}_{ij} \quad \dots \quad \bar{Z}_{mn} \right] \quad (1)$$

where,

$$\bar{Z}_{ij} = R_{ij} + jX_{ij} \quad i, j = 1, \dots, n \quad i \neq j \quad (2)$$

Bus data is given by

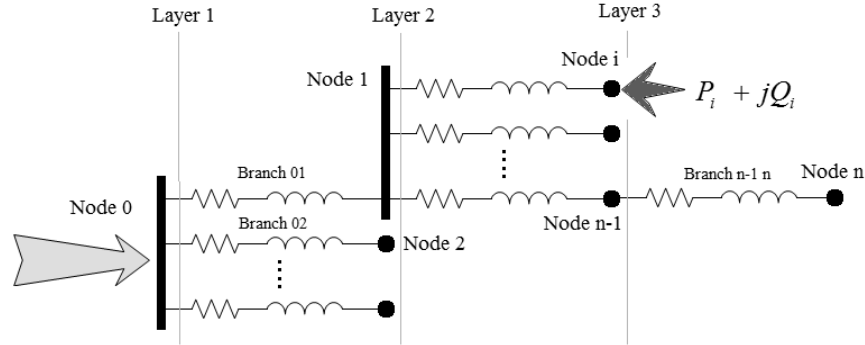


Fig. 1. Branch and node numbering of a radial distribution network

$$\mathbf{S} = \begin{bmatrix} \bar{S}_1 \\ \vdots \\ \bar{S}_i \\ \vdots \\ \bar{S}_n \end{bmatrix} = \begin{bmatrix} P_1 + jQ_1 \\ \vdots \\ P_i + jQ_i \\ \vdots \\ P_n + jQ_n \end{bmatrix} \quad (3)$$

where net nodal active and reactive powers are given by generated and demanded powers:

$$P_i = P_{Gi} - P_{Di} \quad (4)$$

$$Q_i = Q_{Gi} - Q_{Di} \quad (5)$$

The numbering of branches in one layer begins only after all the branches in the previous layer have been numbered. Considering that initial voltages are known: voltage at substation is set $\bar{V}_0 = V_{ref}$ and an initial voltage vector is given by:

$$\mathbf{V}^0 = [\bar{V}_1^0 \quad \dots \quad \bar{V}_i^0 \quad \dots \quad \bar{V}_n^0] \quad (6)$$

The state of the system is reached solving two steps iteratively.

A. Step 1 - Backward Sweep

For each iteration k , branch currents are aggregated from loads to origin:

$$\mathbf{J}^k = \mathbf{T} \cdot \mathbf{I}^k \quad (7)$$

The relationship between nodal currents \mathbf{I}^k and branch currents \mathbf{J}^k is set through an upper triangular matrix \mathbf{T} accomplishing the Kirchhoff Current Laws (KCL) [7]. Each element \bar{I}_i^k of \mathbf{I}^k associated to node i is calculated as function of injected powers \bar{S}_i and its voltage profile \bar{V}_i^k as shown below:

$$\bar{I}_i^k = -\frac{\bar{S}_i^*}{\bar{V}_i^{k*}} \quad i = 1, \dots, n \quad (8)$$

B. Step 2 - Forward Sweep

Nodal voltage vector \mathbf{V} is updated from the origin to loads according the Kirchhoff Voltage Laws (KVL), using previously calculated branch currents vector \mathbf{J} , branch impedances vector \mathbf{Z} :

$$\mathbf{V}^{k+1} = \mathbf{V}_0 - \mathbf{T}^T \cdot \mathbf{D}_Z \cdot \mathbf{J}^k \quad (9)$$

where \mathbf{V}_0 is a n -elements vector with all entries set at voltage at origin (swing node) \bar{V}_0 and branch impedances \mathbf{D}_Z is the diagonal matrix of vector \mathbf{Z} :

C. Convergence

Updated voltages are compared with previous voltages in order to perform convergence check in.

$$\varepsilon \leq |\bar{V}_i^{k+1} - \bar{V}_i^k| \quad i = 1, \dots, n \quad (10)$$

III. ILLUSTRATIVE EXAMPLE: SIMPLY 4-NODE NETWORK

To illustrate the proposed methodology, it is used the 4-node example shown in Fig. 2. Length of all sections is 1 mile. Load demand at nodes 2 and 3 are 2MW with $\cos \varphi = 1.0$.

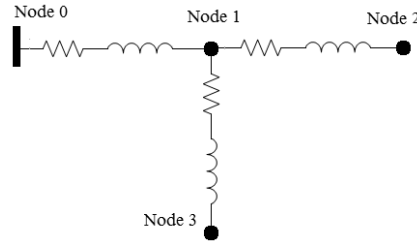


Fig. 2. 4-Node Network Topology

Using the following bases $S_B = 10\text{MW}$ and $V_B = 12.47\text{kV}$, data and results are given in per unit. Loads are 0.2 in nodes 2 and 3. Reference voltage at node 0 is $\bar{V}_0 = 1 + j0$ and initial voltages are set $\mathbf{V}^0 = \begin{bmatrix} 1 + 0j & 1 + 0j & 1 + 0j \end{bmatrix}$.

Branches are represented by:

$$\mathbf{Z} = \mathbf{R} + j\mathbf{X} = \begin{bmatrix} .0296 \\ .0296 \\ .0296 \end{bmatrix} + j \begin{bmatrix} .0683 \\ .0683 \\ .0683 \end{bmatrix}$$

Network topology is represented through a 3x3 upper triangular matrix \mathbf{T} .

$$\mathbf{T} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Then, \mathbf{D}_Z is:

$$\mathbf{D}_Z = \begin{bmatrix} .0296 + j.0683 & .0 & 0 \\ 0 & .0296 + j.0683 & 0 \\ 0 & 0 & .0296 + j.0683 \end{bmatrix}$$

Solution reached at iteration 3 for $\varepsilon = 10^{-4}$ and displayed in Table I. Results are presented in per unit and degrees.

TABLE I
4 NODE STATE OF THE SYSTEM - BALANCED APPROACH

V_0	θ_0	V_1	θ_1	V_2	θ_2	V_3	θ_3
1.000	0.00	0.987	-1.59	0.981	-2.40	0.981	-2.40

IV. CONCLUSION

This paper describes a convenient formulation of the Backward/Forward (BW/FW) Sweep Power Flow applied to radial distribution systems with distributed generation. Proposed formulation was applied in a illustrative test system.

APPENDICES

A. NOMENCALTURE

List of Symbols

\mathbf{D}_Z	Diagonal matrix of branch impedance vector \mathbf{Z}
\mathbf{R}	Diagonal matrix of branch resistance vector $\Re\mathbf{Z}$
\mathbf{X}	Diagonal matrix of branch reactance vector $\Im\mathbf{Z}$
ε	Convergence criteria
\mathbf{I}	Current vector
\mathbf{J}	Branch Current vector \mathbf{J}
n	Number of nodes, excluding origin
\mathbf{P}	Active Power Injected vector
\mathbf{Q}	Reactive Power Injected vector
P_j	Active Power Injected at node j
Q_j	Reactive Power Injected at node j
P_{Dj}	Active Power Demanded at node j
Q_{Dj}	Reactive Power Demanded at node j
P_{Gj}	Active Power Generated at node j
Q_{Gj}	Reactive Power Generated at node j
R_{ij}	Resistance between node i and node j
S_{Dj}	Apparent Power Demanded at node j
S_{Gj}	Apparent Power Generated at node j
\mathbf{T}	Triangular matrix
\mathbf{V}	Voltage vector
X_{ij}	Reactance between node i and node j
\mathbf{Z}	Branch Impedance vector \mathbf{Z}
\bar{Z}^{ij}	Branch Impedance between node i and node j
\mathbf{Z}^{ij}	Impedance matrix between node i and node j

Operators

T	Transpose Matrix
D	Diagonal Matrix
$*$	Conjugate of a complex number

Sub-Indexes

- i Associated to node i
 j Associated to node j
 k Associated to iteration k

REFERENCES

- [1] R. Berg, E.S. Hawkins, and W.W. Pleines, "Mechanized Calculation of Unbalanced Load Flow on Radial Distribution Circuits," *IEEE Transactions on Power Apparatus and Systems*, Volume PAS-86, No. 4, pp.415-421, Apr 1967
- [2] W. H. Kersting and D. L. Mendive, "An application of ladder network theory to the solution of three phase radial load flow problem," in *Proc. IEEE PES Winter Meeting*, 1976, New York, paper A76044-8 (IEEE, New York, 1976).
- [3] D. Shirmohammadi, H.W. Hong, A. Semlyen, and G.X. Luo, "A compensation-based power flow method for weakly meshed distribution and transmission networks," *IEEE Transactions on Power Delivery*, Vol. 3, No. 2, pp.753-762, 1988.
- [4] S. Ghosh and D. Das, "Method for load-flow solution of radial distribution networks," *IEE Proc. Generat. Transm. Distrib.*, Vol. 146, No. 6, pp. 641-648, 1996.
- [5] Y Fukuyama, Y Nakanishi, H-D Chiang "Fast distribution power flow using multi-processors," *Electrical Power & Energy Systems*, Vol. 18, No. 5, pp. 331-337, 1996
- [6] D. Thukaram, H.M.W.Banda, and J. Jerome, "A robust three-phase power flow algorithm for radial distribution systems," *Electric Power Systems Research*, Vol. 50, No. 3, pp. 227-236, 1999.
- [7] S. Jovanovic and F. Milicevic, "Triangular distribution load flow," *IEEE Power Engineering Review*, pp. 60-62, 2000.
- [8] D. Rajicic, R. Taleski, "Two novel methods for radial and weakly meshed network analysis," *Electric Power Systems Research*, Vol. 48, No.2, pp. 79-87, Dec. 1998
- [9] Aravindhabuba, P., Ganapathy, S., and Nayar, K. R., "A novel technique for the analysis of radial distribution systems," *Electric Power Systems Research*, Vol. 23, No. 3, pp. 167-171, 2001.
- [10] Y. Zhu and K. Tomsovic, "Adaptive power flow method for distribution systems with dispersed generation," *IEEE Transactions on Power Delivery*, Vol. 17, No. 3, pp. 822-827, 2002.
- [11] J. Liu, M.M.A. Salama and R.R. Mansour, "An efficient power flow algorithm for distribution systems with polynomial load," *Int. J. Elect. Eng. Educat.*, Vol. 39, No. 4, pp. 371-386, 2002.
- [12] M. Afsari, S. P. Singh G. S. Raju G. K. Rao "A Fast Power Flow Solution of Radial Distribution Networks," *Electric Power Components and Systems*, 30:1065-1074, 2002
- [13] J.H. Teng,"A direct approach for distribution system load flow solutions," *IEEE Transactions on Power Delivery*, Vol. 18, No. 3, pp. 882-887, 2003.
- [14] K, Prasad, N.C. Sahoo, A. Chaturvedi and R. Ranjan. "A simple approach for branch current computation in load flow analysis of radial distribution systems," *Int. J. Elect. Eng. Educat.* Vol. 44 No. 1 pp. 49-63, Jan. 2007
- [15] G.W. Chang, S.Y. Chu and H.L. Wang "An Improved Backward/Forward Sweep Load Flow Algorithm for Radial Distribution Systems," *IEEE Transactions on Power Systems*, Vol. 22, No. 2, pp.882-884, May 2007