Dynamic Ground Effect on the Longitudinal Stability of Airplanes during the Landing

Pedro J. González1 Pedro J. Boschetti2 Elsa M. Cárdenas3 and Miguel Rodriguez-Celi4

Universidad Simón Bolívar, Naiguatá, Edo. Vargas, 1160, Venezuela

The objective of the work presented herein is to study the influence of dynamic ground effect on the longitudinal stability of airplanes during un-power landing by numerical simulation. A method was developed to design a landing control system which considers dynamic ground effect by the inclusion of the $h$ derivatives in the longitudinal equations of motion. A case study was presented to illustrate with an example the influence of dynamic ground effect on the longitudinal stability using the Unmanned Airplane for Ecological Conservation. Open-loop flight simulations were performed considering dynamic ground effect using a program written in FORTRAN language. When the vehicle achieved heights lower than 2.5935 m during the simulations, an unstable response appears. A landing control system was designed using the methodology presented in this paper, and close-loop simulations were completed using a model created in Simulink. The control system drove the airplane to the runway, only when the $h$ derivatives were included in the equations of motion. The unstable response produced by the dynamic ground effect has a strong influence in the flare manoeuvre, and for this reason, in the design of a landing control system, the $h$ derivatives must be included in the longitudinal equations of motion.

Nomenclature

$b$ = wingspan
$C_{Do}$ = viscous drag coefficient or parasite drag coefficient
$C_{Lo}$, $C_{Mo}$ = lift and pitching moment coefficients at zero angle of attack
$C_{Lp}$, $C_{Mp}$ = variation of lift and pitching moment coefficients with pitch rate
$C_{Lu}$, $C_{Du}$, $C_{Mu}$ = variation of lift, drag and pitching moment coefficients with non-dimensional speed
$C_{Lh}$, $C_{Dh}$, $C_{Mh}$ = variation of lift, drag and pitching moment coefficients with $H/\bar{e}$
$C_{Lo}$, $C_{Do}$, $C_{Mo}$ = lift, drag and pitching moment slopes
$C_{Li}$, $C_{Mi}$ = variation of lift and pitching moment coefficients with rate of change of angle of attack
$C_{Li_e}$, $C_{Mi_e}$ = variation of lift and pitching moment coefficients with elevator deflection
$\bar{e}$ = medium chord
$D_e$ = contact distance
d = perpendicular deviation from the glide–path reference
e$, $e_d$ = reference error and deviation error from the glide–path
$GsT$ = glide–slope transmitter
g = gravitational acceleration
$H$ = height of mean aerodynamic chord above ground
$\dot{H}$ = time–varying reference with the altitude
$H_o$ = initial height of the flare
$h$ = non–dimensional height, $H/\bar{e}$
$h_o$ = stick fixed neutral point
$I_y$ = pitching moment of inertia
$K$ = gain vector

1 Assistant Professor, Department of Industrial Technology, Camurí Grande Valley, Member AIAA
2 Aggregate Professor, Department of Industrial Technology, Camurí Grande Valley, Senior Member AIAA.
3 Aggregate Professor, Department of Industrial Technology, Camurí Grande Valley, Senior Member AIAA.
4 Instructor, Department of Industrial Technology, Camurí Grande Valley.
I. Introduction

The $h$ derivatives have a strong influence on the longitudinal dynamic stability of an airplane when it flies near to the ground.\textsuperscript{1} Boschetti and Cárdenas\textsuperscript{1} demonstrated (by dynamic analysis on a light unmanned airplane) that the longitudinal mode of motions, phugoid and subsidence, are considerably affected by ground effect. The $h$ derivatives describe the variation of the coefficients with height above ground\textsuperscript{2} and their values depend on height. The $h$ derivatives could be associated with dynamic ground effect on airplanes. Staufenbiel,\textsuperscript{3} Boschetti \textit{et al}\textsuperscript{4} and Chun and Chang\textsuperscript{5} established some nonlinear effects on airplanes or ekranoplans flying near to the ground when the $h$ derivatives are included in the dynamic equations of motion.

Staufenbiel and Schlichting\textsuperscript{6} explained the interaction between the $h$ derivatives (or height-dependent aerodynamic coefficients) and longitudinal stability and their importance in flare maneuvers. Then, González\textsuperscript{7} and González \textit{et al}\textsuperscript{8} demonstrated via simulation that an airplane is capable of making the touchdown due to the unstable response of the airplane caused by dynamic ground effect. However, some authors\textsuperscript{9,10} neglected the $h$ derivatives in automatic landing system design, and they vary the stability coefficients values with height above ground.

The inclusion of the $h$ derivatives in the equations of motion used to design a landing control system and simulations could consider the dynamic ground effect during landing. The objective of the work presented herein is to study the influence of dynamic ground effect on the longitudinal stability of airplanes during un-power landing by numerical simulation.
II. Longitudinal Dynamic Stability in Ground Effect

Equation (1) presents the longitudinal system of linear equations of motion used to describe the effect of variations of height above ground considering ground effect obtained by Boschetti and Cárdenas.1 This system was obtained from a classical system of equations presented by Etkin which considers atmospheric density variations and ground effect.2 Table 1 shows the derivatives expressions presented in Eq. (1).

$$
\begin{bmatrix}
\Delta \dot{u} \\
\Delta \dot{v} \\
\Delta \dot{q} \\
\Delta \dot{\theta} \\
\Delta \dot{H}
\end{bmatrix} =
\begin{bmatrix}
X_u & X_v & 0 & -g & X_h \\
Z_u & Z_v & u_a & 0 & Z_h \\
M_u + M_w Z_u & M_w Z_v & M_q & M_u u_a & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & -1 & 0 & u_o & 0
\end{bmatrix}
\begin{bmatrix}
\Delta u \\
\Delta v \\
\Delta q \\
\Delta \theta \\
\Delta H
\end{bmatrix}
$$

(1)

If in Eq. (1) the \(h\) derivatives are equal to zero, this system of equations becomes the classical system of equations for longitudinal mode in free flight condition. Five eigenvalues are obtained from the solution of the system of equations shown in Eq. (1). The short-period and the phugoid modes are represented by two complex pairs, and the subsidence mode is represented by one real root. The subsidence mode describes the vehicle’s capability to maintain its initial height in time.5

### Table 1. Summary of longitudinal derivatives.

<table>
<thead>
<tr>
<th>Derivative</th>
<th>S.I. Unit</th>
<th>Derivative</th>
<th>S.I. Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>(X_u = -(C_{Da} + 2 C_{Do}) (u_a) (QS/m))</td>
<td>s(^{-1})</td>
<td>(Z_{ik} = -C_{ik} QS/m)</td>
<td>m/s(^2)</td>
</tr>
<tr>
<td>(X_v = -(C_{Da} - C_{Do}) QS/(m \cdot u_a))</td>
<td>s(^{-1})</td>
<td>(M_q = C_{Mq} \bar{c}/(2u_a) (QS \bar{c}/I_y))</td>
<td>s(^{-1})</td>
</tr>
<tr>
<td>(X_h = -C_{Ma} (l/\bar{c}) (QS/m))</td>
<td>s(^{-1})</td>
<td>(M_u = C_{Ma} (l/u_a) (QS \bar{c}/I_y))</td>
<td>(m(\cdot s))(^{-1})</td>
</tr>
<tr>
<td>(X_{ik} = -C_{Dik} (QS/m))</td>
<td>m/s(^2)</td>
<td>(M_u = C_{Mik} QS \bar{c}/(I_j u_a))</td>
<td>(m(\cdot s))(^{-1})</td>
</tr>
<tr>
<td>(Z_u = -(C_{Da} + 2 C_{Do}) (u_a) (QS/m))</td>
<td>s(^{-1})</td>
<td>(M_q = C_{Mik} \bar{c}/(2u_a) QS \bar{c}/(I_j u_a))</td>
<td>m(^{-1})</td>
</tr>
<tr>
<td>(Z_v = -(C_{Da} + C_{Do}) QS/(m \cdot u_a))</td>
<td>s(^{-1})</td>
<td>(M_k = C_{Mik} (l/\bar{c}) QS \bar{c}/(I_j))</td>
<td>(m(^{-1}))(^{-1})</td>
</tr>
<tr>
<td>(Z_h = -C_{ik} (l/\bar{c}) (QS/m))</td>
<td>s(^{-1})</td>
<td>(M_k = C_{Mik} QS \bar{c}/(I_j))</td>
<td>s(^{-2})</td>
</tr>
</tbody>
</table>

III. Landing Control System

A conventional automatic landing system uses a radio signal directed to the glide–path from the ground at approximately 2.5 to 3 degrees. The equipment on board the aircraft must be able to compute the perpendicular displacement of the aircraft from the signal describing the glide–slope and to measure the angular deviation from the signal. Figure 1 describes the descending trajectory of an airplane with a velocity \(u_o\) and the glide–slope angle \(\gamma\). The reference trajectory has an angle \(\gamma_r\), with glide–slope transmitter in the indicated point \(GkT\). The landing control system has to maneuver the airplane to reduce to zero the glide–path deviation. Then, when the proper height is achieved, the reference changes to perform the flare, and it finishes with the touchdown. The model developed by González7 uses a multi-input/multi-output system to solve the problem; this control theory allows computing all the control variables at the same time. It is able to solve the dynamic system presented in Eq. (1) using the optimum control theory.

![Figure 1. Description of the landing phase.](image-url)
A. Glide–Slope Coupler

The landing control equipment measures the angular deviation $\delta$ and the range to the transmitter, and then it calculates $d$ by Eq. (2), when the aircraft trajectory is related to the perpendicular deviation for control design.

$$d = R \cdot \sin \delta$$

Figure 1 shows the component of the velocity perpendicular to the glide–path, given by

$$\dot{d} = u_o \cdot \sin(\gamma - \gamma_r)$$

The signal should guide the aircraft through the glide–path, and the only control available to complete the landing is the elevator input. The dynamic states presented in Eq. (1) are modified to include the elevator deflection; then, the state and the control input are

$$x = [\Delta u \quad \Delta \dot{w} \quad \Delta q \quad \Delta \theta \quad \Delta H \quad \Delta \delta_y]^T$$

$$\eta = [\eta_e]$$

The control problem is described by Eq. (5), where $A$ is the state matrix and $B$ is the control vector.

$$\dot{x} = Ax + B \eta$$

Previously, the perpendicular deviation from the glide–path reference was defined by the glide–path geometry. Placing the variables in terms of the state vector, Eq. (3) can be expressed as

$$\dot{d} = u_o \cdot \theta - \frac{u_o}{57.2958} \gamma_r$$

With $\gamma_r$ in degrees and $\theta$ in radians.

The perpendicular glide–path distance can be included as a state

$$x = [\Delta u \quad \Delta w \quad \Delta q \quad \Delta \theta \quad \Delta H \quad d \quad \Delta \delta_y]^T$$

This control problem was adapted by González,\textsuperscript{7} based on the principles established by Blakelock\textsuperscript{12} and the procedures developed by Stevens and Lewis.\textsuperscript{11} The system must reduce $d$ to zero. Figure 2 shows a tracking problem with reference commands $r_d$ equal zero and the tracking error $e_r = [e_d]$ with

$$e_r = r_d - d$$

To reduce the deviation of the airplane from the glide–path, it is necessary the implementation of a compensator. The compensator helps the control gain to hold $d$ as small as possible, without exceeding the physical limitations of the elevator. The proposed compensator is

$$\frac{w_d}{e_d} = \frac{K_{(1)}}{s(s + S_1)} + \frac{K_{(2)}}{s} + K_{(3)}$$

The distance compensator pole is $Tf_1$.\textsuperscript{11} The compensator variables are represented as state variables to obtain their gains by linear quadratic regulator (LQR). The transfer functions are presented in Eq. (10).

$$Tf_1 = \frac{1}{s}; \quad Tf_2 = \frac{1}{s + S_d}; \quad Tf_3 = \frac{S_d}{s + S_d}$$

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Equation (11) represents the compensator error, which is integrated to obtain the error controller shown in Eq. (12).

\[ \dot{e}_d = e_d = -d + r_d \]  

(11)

\[ \dot{x}_d = -S_x \cdot x_d + \epsilon_d \]  

(12)

The resultant control compensator input is given by Eq. (13).

\[ \eta_e = -K_{(1)} \cdot x_d - K_{(2)} \cdot \epsilon_d - K_{(3)} \cdot (r_d - d) \]  

(13)

The compensator is included in the system described by Eq. (5), and the state vector is expressed as

\[ x = [\Delta u \ \Delta w \ \Delta q \ \Delta \theta \ \Delta H \ d \ \Delta \delta_e \ x_d \ \epsilon_d]^T \]  

(14)

Then, the resultant augmented system is defined by Eq. (15)

\[ \dot{x} = Ax + B \eta + Gr \]  

(15)

with

\[ A = \begin{bmatrix}
X_u & X_w & 0 & -g & X_h & 0 & X_{\delta_e} & 0 & 0 \\
Z_u & Z_w & u_o & 0 & Z_h & 0 & Z_{\delta_e} & 0 & 0 \\
M_u + M_u Z_u & M_w + M_w Z_u & M_q + M_q u_o & 0 & M_h + M_h Z_h & 0 & M_{\delta_e} + M_{\delta_e} Z_{\delta_e} & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & u_o & 0 & 0 & 0 & 0 & 0 \\
0 & -1/57.2958 & 0 & u_o/57.2958 & 0 & 0 & -S_d & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -S_x & 1 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
\end{bmatrix} \]  

(16)
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\[
B = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & u_r/57.2958 & S_{\delta} & 0 & 0 \\ \end{bmatrix} \quad ; \quad G = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ \end{bmatrix}
\]

(17)

It is necessary to define the augmented input to include the constant disturbance \( \gamma_r \) in Eq. (6),

\[
\eta' = [\eta, \gamma_r]^T
\]

(18)

The reference input considered in the tracking problem is \( r = [r_\delta] \). The variable \( \gamma_r \) is considered an exogenous input, which is constant during the descent of the aircraft. Then,

\[
y = Cx + Fr
\]

(19)

with

\[
C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \end{bmatrix} ; \quad F = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ \end{bmatrix}
\]

(20)

The control input is defined by the output feedback as

\[
\eta = [\eta_r] = -K \cdot y
\]

(21)

B. Flare control system

The flare control system is based on the model proposed by Stevens and Lewis.\textsuperscript{11} At a given height, the control system is activated by adjusting the rate of descent in order for the airplane to complete the landing. Figure 3 illustrates the flare maneuver.

Like the glide–slope coupler, the flare control system works as a tracking system, tracking a time-varying reference with the altitude \( H \). The commanded altitude is defined by Eq. (22), as an exponential function.\textsuperscript{11}

Figure 3. Flare maneuver diagram.
The initial conditions of the problem of the flare are $H_0$, $\tau$ and $D_c$, and these are chosen based on the operator requirements.\textsuperscript{11}

Equation (23) describes the distance to the glide–slope transmitter.

$$ R = \frac{H_o}{\tan(\gamma)} \tag{23} $$

Then, the initial height is expressed as

$$ H_o = u_e \sin(\gamma) - D_c f(3u_e) \tag{24} $$

When the aircraft reaches the flare initial height, the switch in Fig. 2 changes the glide–path exogenous input to the commanded altitude function, the state vector changes, the flare compensator is used to reduce the difference with $H$ and the variable $d$ is replaced in Eq. (14) by the reference altitude, as seen in Eq. (25).

$$ x = \begin{bmatrix} \Delta u \\ \Delta w \\ \Delta q \\ \Delta \theta \\ \Delta H \\ \Delta \delta_e \\ x_f \end{bmatrix} \tag{25} $$

The state feedback problem is described as

$$ x = Ax + Bu + Gr $$

$$ y = Cx + Fr \tag{26} $$

The flare compensator is similar to the glide–slope coupler compensator, and the error reduction is:

$$ e_r = H - H \tag{27} $$

\section*{IV. Case Study}

A case study is presented to illustrate with an example the influence of dynamic ground effect on the longitudinal stability. The aircraft selected is the unmanned airplane for ecological conservation (ANCE), because, for this airplane, a complete aerodynamic database at different heights above ground was recently obtained by panel methods.\textsuperscript{1} Tables A1 and A2 present the general characteristic of this airplane, and the aerodynamic and stability coefficients at different heights. In a previously published work,\textsuperscript{1} it was demonstrated that the airplane is longitudinally dynamically stable between free flight and $H = 2.5935$ m ($H/b = 0.5$), and unstable for lower heights.

A program to perform flight simulations\textsuperscript{13} written in Visual FORTRAN\textsuperscript{14} was used to calculate the open-loop response of the airplane flying near to the ground in time-domain. The program solves the system of linear equations of motion shown in Eq. (1) in state space form with a four-order Runge-Kutta method. The discrete values of the stability coefficients shown in Table A2 were used to create interpolated functions, which are used for the program to obtain the stability coefficients at each altitude. When the altitude changes during a simulation, the program estimates anew the stability derivatives using the corresponding stability coefficients, air density and wind speed.\textsuperscript{13}

Simulations were performed placing ten heights above ground as starting point, with initial speed of 32.0208 m/s (1.3 of stall speed) and time step of 0.015 s. The airplane model was perturbed by elevator deflection of 5 deg step function during 0.2 s in each simulation. Figure 4 illustrates the airplane height above ground in time-domain; the starting heights are shown in the captions. It is important to inform that the minimum value for height above ground is 0.788 m, because this is the distance between the wing and the airplane’s tire tread.

It is observed in Fig. 4 that for starting heights equal to or lower than 2.5935 m, the airplane sinks to the ground due to unstable response; for 2.9987 m, the aircraft sinks to 2.4912 m, where it is unstable and presents a divergent response, until the airplane achieves the stall speed. For greater heights, the aircraft shows a convergent and damped response. These observations correspond with the comments made by Boschetti and Cárdenas\textsuperscript{1} about the longitudinal stability of the airplane near to the ground.
A. Landing control system

Continuing the case study, an automatic landing control was designed for the ANCE using the methodology previously stated in this paper. To accomplish this aim, the state feedback gain for Eq. (21) was initially estimated by the routine \textit{lqry} from Matlab\textsuperscript{®}\textsuperscript{15} using the state matrix Eq. (16). This routine solves the linear quadratic regulator problem in continuous time and the associated Ricatti equation.\textsuperscript{16} The weighting matrices were matched to an identity matrix to prevent the system from becoming unstable.

Figure 5 illustrates a simplified block diagram used for Simulink (a Matlab\textsuperscript{®} application) to make close-loop simulations of the airplane. This includes the references for the glide–slope and the flare. The exogenous references are presented as tracking commands, and they change when the aircraft reaches the initial height of the flare. Each simulation finishes when the tires of the airplane touch the ground.

In order to reduce the error with the references, the gain vector was tuned with Matlab\textsuperscript{®}. The routine \textit{fmincon} was used to obtain the minimum of constrained nonlinear multivariable function. The outputs \textit{d–H, δ, θ} were the minimized states. This tuned output was processed in a different “.m” file, where the variation margins were placed to obtain the \(ΔK\). The resultant \(K\) for the entire landing is given in Eq. (28).

\[
\begin{bmatrix}
0.134652895 \\
0.2103320175 \\
-5.24968760 \\
62.0176348018 \\
0.00000398762 \\
59.2158125601 \\
3.36556067739 \\
0.0000004758 \\
1.0049875573
\end{bmatrix}
\]

Equations (15), (19) and (26) were integrated by the routine \textit{ode45} from Matlab\textsuperscript{®}. This solves ordinary differential equations using the Runge-Kutta-Fehlberg method. The relative tolerance used in this simulation is 0.001. The simulations were performed using an initial velocity of 52.21 m/s and a starting height of 500 m over the ground. The initial condition vector is

\[
\begin{bmatrix}
u_o \\
0 \\
0 \\
\text{H} \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

Equations (15), (19) and (26) were integrated by the routine \textit{ode45} from Matlab\textsuperscript{®}. This solves ordinary differential equations using the Runge-Kutta-Fehlberg method. The relative tolerance used in this simulation is 0.001. The simulations were performed using an initial velocity of 52.21 m/s and a starting height of 500 m over the ground. The initial condition vector is

\[
\begin{bmatrix}
u_o \\
0 \\
0 \\
\text{H} \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

Figure 6 presents the airplane height in time-domain showing the flight–path during ground approach and landing. It can be observed that the automatic control system carried the airplane to the runway.
Figure 7 presents a comparison between the flight-path described by the airplane when is simulated using the longitudinal system of equations of motion expressed in Eqs. (1) and (16), and that one when the vehicle is simulated neglecting the $h$ derivatives. If the $h$ derivatives are not included, the aircraft does not complete the touchdown. Figures 8 and 9 show the variation of pitch rate and the variation of elevator deflection, respectively, in time-domain. When the $h$ derivatives are included, an unstable response at heights lower than 2.5935 m ($H/b = 0.5$) is appreciated. The simulation of the model that excludes the $h$ derivatives does not present unstable response. These observations agree with the conclusions stated by Staufenbiel and Schlichting, and González et al.

V. Conclusion

This paper presents a study to demonstrate the influence of dynamic ground effect on the longitudinal stability of airplanes during un-power landing by numerical simulation. A method to design a landing control system that considers dynamic ground effect by the inclusion of the $h$ derivatives in the longitudinal equations of motion was developed herein.
The tracking problem approach presented in this paper uses the same state matrix for the entire landing problem, thus, a gain vector can control all the flight phases, and consequently, some problems as multiple gain vector estimation and gain scheduling can be avoided.

A case study was presented to illustrate with an example the influence of dynamic ground effect on the longitudinal stability, using the Unmanned Airplane for Ecological Conservation. Open-loop flight simulations were performed by a program written in FORTRAN language, demonstrating that when the vehicle achieved heights lower than 2.5935 m \((H/b=0.5)\) during the simulations, an unstable response appears. For greater heights, the aircraft shows a convergent and damped response. A specific aircraft can be dynamically unstable below a certain height value due to the ground effect.

A landing control system was designed using the methodology presented in this paper. Close-loop simulations were completed using a model created in Simulink achieving that the control system drove the airplane to the runway, only when the \(h\) derivatives were included in the equations of motion. The unstable response produced by the dynamic ground effect has a strong influence in the flare maneuver, and for this reason, in the design of a landing control system the \(h\) derivatives in the longitudinal equations of motion must be included.

**Appendix A**

Table A1 presents the general characteristics of the Unmanned Airplane for Ecological Conservation (ANCE), and Table A2 shows the longitudinal stability coefficients of the vehicle obtained by Boschetti and Cárdenas.¹

| Table A1. General characteristics of the Unmanned Airplane for Ecological Conservation.¹ |
|---------------------------------|---|
| \(b\) | 5.187 m |
| \(\bar{c}\) | 0.604 m |
| \(S\) | 3.1329 \(\text{m}^2\) |
| \(m\) | 182.055 kg |
| Gravity center of the airplane | 0.25 \(\bar{c}\) |
| \(I_x\) | 400 \(\text{kg} \cdot \text{m}^2\) |
| \(C_{Do}\) | 0.0266 |

| Table A2. Longitudinal stability coefficient of the ANCE at different heights.¹ |
|---------------------------------|---|
| \(H\) | 0.7882 | 1.0374 | 1.2968 | 1.5561 | 2.0748 | 2.5935 | 2.9987 | 5.5922 | 8.1857 | 10.779 | 13.373 | \(\infty\) |
| \(H/b\) | 0.152 | 0.2 | 0.25 | 0.3 | 0.4 | 0.5 | 0.5781 | 1.0781 | 1.5781 | 2.0781 | 2.5781 | \(\infty\) |
| \(k\) | 0.0366 | 0.0403 | 0.0427 | 0.0443 | 0.0461 | 0.0471 | 0.0475 | 0.0485 | 0.0487 | 0.0488 | 0.0488 | 0.0488 |
| \(C_{Lo}\) | 0.3392 | 0.3366 | 0.3343 | 0.3326 | 0.3305 | 0.3293 | 0.3288 | 0.3276 | 0.3274 | 0.3273 | 0.3273 | 0.3273 |
| \(C_{Mo}\) | -0.009 | -0.009 | -0.009 | -0.009 | -0.009 | -0.009 | -0.009 | -0.009 | -0.009 | -0.009 | -0.009 | -0.009 |
| \(C_{Da}\) | 0.1478 | 0.1562 | 0.1613 | 0.1645 | 0.168 | 0.1699 | 0.1706 | 0.1726 | 0.173 | 0.1731 | 0.1731 | 0.1731 |
| \(C_{La}\) | 5.953 | 5.7582 | 5.6494 | 5.5806 | 5.5119 | 5.4775 | 5.4603 | 5.4316 | 5.4259 | 5.4202 | 5.4202 | 5.4202 |
| \(C_{L\alpha e}\) | 0.6474 | 0.6417 | 0.6417 | 0.6417 | 0.6417 | 0.6417 | 0.636 | 0.636 | 0.636 | 0.636 | 0.636 | 0.636 |
| \(C_{Ma}\) | 16.97 | 19.43 | 18.71 | 19.3 | 19.06 | 19.06 | 18.95 | 18.93 | 18.93 | 18.93 | 18.93 | 18.93 |
| \(h_o\) | 0.932e | 0.946c | 0.954c | 0.957c | 0.96c | 0.959c | 0.959c | 0.957c | 0.958c | 0.958c | 0.958c | 0.958c |
| \(C_{lh}\) | -0.006 | -0.005 | -0.004 | -0.002 | -0.001 | -7E–04 | -3E–04 | -5E–05 | -2E–05 | 0 | 0 | 0 |
| \(C_{lah}\) | -0.008 | -0.004 | -0.003 | -0.001 | -2E–04 | 0 | 2E–05 | 2E–05 | 2E–05 | 0 | 0 | 0 |

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