Stability and Performance of a Light Unmanned Airplane in Ground Effect

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The objective of the present work is to understand the effect of ground proximity on the aerodynamic performance and stability of a light unmanned aerial vehicle. The flowfield around the airplane was computed by PAN AIR and Athena Vortex Lattice. The ground effect was simulated using the method of images. The stability coefficients and other aerodynamic characteristics were obtained at different heights above ground and in free flight. The results demonstrate that the airplane is lateral-directional statically stable, longitudinal statically stable in free flight and longitudinal statically unstable in ground effect. The dynamic stability characteristics of the airplane were obtained at different heights above ground. The phugoid mode is considerably influenced by ground effect and a divergent and non-oscillatory mode appears when the airplane is near to the ground. This is called non-dimensional height mode. The short-period, the Dutch roll, the roll and the spiral modes are slightly affected by ground effect. Significant differences were obtained when the $z$ derivatives were neglected in the dynamic analysis for longitudinal motion. The present work demonstrates that the performance and stability of the unmanned airplane are considerably influenced by ground effect.

Nomenclature

$AS$ = acceleration sensitivity
$b$ = wingspan
$CAP$ = control anticipation parameter
$C_{Di}$ = induced drag coefficient
$C_{Do}$ = viscous drag coefficient or parasite drag coefficient
$C_L$ = lift coefficient
$C_{max}$ = maximum lift coefficient
$C_{Lw}$, $C_{Mw}$ = lift and pitching moment coefficient at zero angle of attack
$C_{Lp}$, $C_{Mp}$ = variation of lift, and pitching moment coefficients with pitch rate
$C_{Ls}$, $C_{Ds}$, $C_{Ms}$ = variation of lift, drag, and pitching moment coefficients with non-dimensional speed
$C_{Lh}$, $C_{Dh}$, $C_{Mh}$ = variation of lift, drag, and pitching moment coefficients with non-dimensional height
$C_{Ld}$, $C_{Md}$ = lift, drag, and pitching moment slopes
$C_{Lm}$, $C_{Mm}$ = variation of lift, and pitching moment coefficients with rate of change of angle of attack
$C_{Le}$, $C_{Me}$ = variation of lift, and pitching moment coefficients with elevator deflection
$C_{lp}$, $C_{mp}$, $C_{yp}$ = variation of rolling, yawing and side force coefficients with roll rate
$C_{lr}$, $C_{mr}$, $C_{yr}$ = variation of rolling, yawing and side force coefficients with yaw rate
$C_{lp}$, $C_{mp}$, $C_{yp}$ = variation of rolling, yawing and side force coefficients with sideslip angle

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\[ C_{\ell\delta}, C_{n\delta}, C_{Y\delta} = \text{variation of rolling, yawing and side force coefficients with rudder angle} \]
\[ C_{\ell\alpha}, C_{n\alpha}, C_{Y\alpha} = \text{variation of rolling, yawing and side force coefficients with aileron angle} \]
\[ c = \text{medium chord} \]
\[ e = \text{Oswald efficiency factor} \]
\[ e^n = \text{criterion for transition} \]
\[ G = \text{drag reduction factor} \]
\[ g = \text{gravity acceleration} \]
\[ I_x, I_y, I_z = \text{rolling, pitching and yawing moments of inertia} \]
\[ h = \text{height of mean aerodynamic chord above ground} \]
\[ h_0 = \text{stick fixed neutral point} \]
\[ HS = \text{static height stability} \]
\[ k = \text{lift dependent drag factor or induced drag factor, } k = (\pi e R_A)^{-1} \]
\[ L, M, N = \text{dynamic derivatives about rolling, pitching and yawing moment} \]
\[ m = \text{aircraft mass} \]
\[ n = \text{fixed parameter for two-dimensional free transition criterion} \]
\[ p, q, r = \text{roll, pitch and yaw rate} \]
\[ Q = \text{dynamic pressure} \]
\[ R_A = \text{wing aspect ratio} \]
\[ S = \text{wing surface} \]
\[ T = \text{period} \]
\[ t_{1/2}, N_{1/2} = \text{time and number of cycles to-half amplitude} \]
\[ t_2 = \text{time to-double amplitude} \]
\[ u, w = \text{longitudinal and vertical components of velocity} \]
\[ u_o = \text{initial forward speed} \]
\[ X, Y, Z = \text{dynamic derivatives about forces } x, y \text{ and } z \text{ orientations} \]
\[ z_e = \text{non-dimensional height above ground, } z_e = H/b \]
\[ \beta = \text{sideslip angle} \]
\[ \delta_e, \delta_r, \delta_a = \text{elevator, rudder and aileron deflection} \]
\[ \zeta = \text{damping ratio} \]
\[ \theta, \phi = \text{pitch and bank angle} \]
\[ \theta_o = \text{initial pitch angle} \]
\[ \sigma^{-1} = \text{maximum time constant} \]
\[ \omega_n = \text{undamped natural frequency} \]

**Subscripts**

\[ OGE = \text{out-of-ground effect} \]
\[ p, q, r = \text{derivative with respect to roll, pitch and yaw rate} \]
\[ u, w = \text{derivative with respect to longitudinal and vertical components of velocity} \]
\[ w = \text{derivative with respect to rate of change of vertical velocity} \]
\[ \beta = \text{derivative with respect to sideslip angle} \]
\[ \delta_e, \delta_r, \delta_a = \text{derivative with respect to elevator, rudder and aileron deflection} \]

**I. Introduction**

**D**uring take-off and landing, the distances between the airplane and the ground are relatively small in comparison with the dimensions of the aircraft, and the airplane aerodynamic coefficients are influenced by the ground proximity. The three-dimensional wing flowfield is considerably affected by ground effect. This phenomenon was early studied by Wieselberger, who achieved a simple model based on Prandtl’s multiplanes theory to determine the influence of ground in induced drag. More recently, Suh and Ostowari and Laitone obtained models to estimate the drag reduction factor due to ground effect. The static and dynamic stability of airplanes in ground effect was clearly stated by Staufenbiel and Schlichting.

The ground effect is essentially a non-viscous problem, and a satisfactory solution could be achieved by potential flow methods. For this reason, panel methods (including the vortex lattice method) are widely used to study ground effects in complete airplane configurations. Symes achieved good agreement between wind tunnel data and values computed by a panel method code for lift coefficient in a three-dimensional configuration. Curry and Owens
combined flight tests, wind tunnel tests, and the vortex lattice method to improve understanding of the ground effect characteristics of the supersonic transport Tupolev Tu-144.9

The aim of the present research is to understand the effect of ground proximity on the aerodynamic performance and stability of light unmanned aerial vehicles by means of the Unmanned Airplane for Ecological Conservation (ANCE, for its Spanish acronym). The dynamic stability, aircraft response and flying qualities are especially interesting in this paper. The ANCE is a monoplane twin-boom, pusher–propeller airplane with a rectangular wing of 3.13 m² of surface area, 5.187 m of wingspan, and a wing aspect ratio of 8.57, with a maximum take-off mass of 182.055 kg, and with the center of gravity at 0.25c. The aircraft was originally designed to look for oil leakages from offshore facilities and transporting pipelines at 41.18 m/s at 2,438 m above sea level, for a wing Reynolds number of $1.413 \times 10^6$.10 Figure 1 shows a three-dimensional view of the ANCE.

In order to understand the aerodynamic performance of the ANCE, early wind tunnel test,12,13 and computational fluid dynamic simulation based on potential flow theory14 had been carried out to date. The stability and flying qualities had been studied using the vortex lattice method.15 Some modifications had been done over the original design in order to reduce drag.11-14

II. General Characteristics of Wings and Airplanes in Ground Proximity

The ground effect on the airplanes is a complex phenomenon. The complete flowfield is modified by the ground plane. Generally, when a wing approaches the ground, its lift curve slope increases and the induced drag decreases. The trailing vortices of a wing are reflected in the ground plane. This reduces the strength of the vortices16 and the image vortices induce an up-wash at the real wing so that the total induced incidence for a wing in ground proximity is smaller than in an unbounded stream.17 The induced drag coefficient for a wing of large aspect ratio ratio flying in proximity to the ground is equal to $G k C_L^2$. The drag reduction factor describes how the induced drag is affected by the ground effect. In free flight, $G$ is equal to one. This concept was introduced by Wieselberger1 who presented an expression of drag reduction factor based on a previous Prandtl work.2 In this paper, more exact expressions of $G$ were extracted from Suh and Ostowari3 and Laitone4, and these are shown in Eqs. (1) and (2), respectively.

$$G = 1 - \left(2e/\pi^2\right) \ln \left[1 + \left(\pi/4\right)(2z_e)^2\right]$$ \hspace{1cm} (1)

$$G = \left[1 - (2/\pi + (16/\pi \cdot z_e)^2)\right] / \left[1 + (16/\pi \cdot z_e)^2\right]$$ \hspace{1cm} (2)

A. Effects on Airplanes Stability

While the lateral-directional stability is not expected to be significantly influenced by ground proximity, the longitudinal stability is considerably affected by the ground effect. The downwash angle value at the tail in a conventional airplane in ground effect is less than the one in free flight. This augments the lift curve slope of the horizontal tail, and the tail effectiveness is increased, and the airplane becomes more stable.16

The condition of “static height stability” presented by Staufenbiel and Schlichting,5 is shown in Eq. (3). This represents the variations of static stability when the aircraft approaches the ground.

$$HS = C_{Lz} - (C_{Mc}/C_{Ma}) \cdot C_{Ma} < 0$$ \hspace{1cm} (3)

The equations of motion which describe the longitudinal dynamic response of an airplane in ground effect are formulated in state space as $x' = Ax + B\eta$, where $x = [\Delta u, \Delta w, \Delta q, \Delta \theta, \Delta z_e]^T$ is the state vector, $\eta = [\Delta \delta_e]^T$ is the control vector, and the matrices $A$ and $B$ contain the stability and control derivatives. This system of equations was original presented by Etkin18 to describe the effect of variations of height above ground in the longitudinal mode. Atmospheric density variations and ground effect were considered by Etkin to express the “height above ground”18.
In this work, Etkin's original system of equations was adapted to consider only ground effect. Equations (4) and (5) present matrices $A$ and $B$.

\[
A = \begin{bmatrix}
X_u & X_w & 0 & -g & X_z \\
Z_u & Z_w & u_o & 0 & Z_z \\
M_u + M_w Z_u & M_w + M_q Z_u & M_q + M_w u_o & 0 & M_z + M_q Z_z \\
0 & 0 & 1 & 0 & 0 \\
0 & u_o & 0 & -u_o & 0 \\
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
X_{\delta_e} \\
Z_{\delta_e} \\
M_{\delta_e} + M_w Z_{\delta_e} \\
0 \\
0 \\
\end{bmatrix}
\]

Matrix $A$ has five eigenvalues: two complex pairs associated with two oscillatory modes, the short-period and the phugoid, and one is real representing a first order mode. This system of equations considers the variations of height above ground using the $z$ derivatives. When these derivatives are zero, the system of equations becomes the classical equation of motion in free flight.

The lateral-directional equations of motion are formulated in state space $x'=Cx+D\eta$, where $x=[\delta \beta, \delta \rho, \delta \varphi]^T$ is the state vector, and $\eta=[\Delta \delta \beta, \Delta \delta \rho, \Delta \delta \varphi]^T$ is the control vector. No consideration was taken about ground effect for the lateral-directional mode, as it is not expected that perturbations in this mode affect the non-dimensional height. The ground effect influences this mode by the potential change in the derivatives caused by the ground proximity. Equations (6) and (7) express matrices $C$ and $D$, respectively.

\[
C = \begin{bmatrix}
\frac{Y_{\beta}}{u_o} & \frac{Y_{\rho}}{u_o} & -\left(1 - \frac{Y_{\varphi}}{u_o}\right) \frac{g \cos \theta_o}{u_o} \\
L_{\beta} & L_{\rho} & L_{\varphi} & 0 \\
N_{\beta} & N_{\rho} & N_{\varphi} & 0 \\
0 & 1 & 0 & 0 \\
\end{bmatrix}
\]

\[
D = \begin{bmatrix}
0 & \frac{Y_{\beta}}{u_o} \\
L_{\delta_e} & L_{\delta_e} \\
N_{\delta_e} & N_{\delta_e} \\
0 & 0 \\
\end{bmatrix}
\]

From matrix $C$, four eigenvalues are obtained: a pair of complex roots which represents the Dutch Roll mode, and two real roots for roll and spiral modes. Appendix A shows the mathematical expressions of dimensional derivatives contained in matrices $A$, $B$, $C$ and $D$.

### III. Aerodynamic Analysis Methods

The Panel Aerodynamics (PAN AIR)\(^{20}\) and the Athena Vortex Lattice (AVL)\(^{21}\) open source codes were used to compute the fluid flowfield around the airplane with and without ground effect.

PAN AIR is a high-order panel method code capable of solving a variety of boundary value problems in steady subsonic or supersonic inviscid flow by the classic three–dimensional Prandtl–Glauert equation for linearized compressible flow. The configuration is represented by a distribution of linear source and quadratic double singularities, each of which is a solution of the Prandtl–Glauert equation. The singularity strength parameters are determined by solving the appropriate boundary condition equations. Once these are known, the velocity and

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potential fields are computed. The pressure field can then be calculated from an appropriate pressure–velocity relationship, and forces and moments calculated by pressure integration. In order to improve the accuracy of the induced drag calculation, the version A502i can calculate it by Trefftz plane analysis. The PAN AIR pilot code and later versions have been used and validated to compute aerodynamic forces and moments, and pressure distribution on arbitrary configurations in free flight, and validated against experimental data to calculate the influence of ground proximity on a single wing. A complete discussion of the method may be found in Ref 27.

The Athena Vortex Lattice is a code capable of solving the non-viscous flowfield around the aerodynamic configurations which consists mainly of thin lifting surfaces at small angles of attack and sideslip. The lifting surfaces are represented as single-layer vortex sheets, discretized into horseshoe vortex filaments. At a specific control point, the velocities induced by each horseshoe vortex are calculated by the law of Biot–Savart. A set of linear algebraic equations for the horseshoe-vortex strengths is obtained when all control points on the wing are summed, satisfying the boundary condition of no flow through the wing. The wing circulation and the pressure differential between the upper and lower surfaces are connected to the vortex strengths. The forces are obtained by integration of the pressure differentials. AVL could also model slender bodies via source and doublet filaments, and force and moment predictions are consistent with slender-body theory. Athena Vortex Lattice assumes quasi-steady flow and treats the compressibility using the Prandtl-Glauert transformation.

To verify the capability of PAN AIR and Athena to simulate ground effect, a rectangular wing with similar aspect ratio and airfoil section of the ANCE wing with no twist was tested using both codes at eight different distances above ground (\(z_e\) equal to 0.1, 0.16667, 0.25, 0.5, 1.0, 1.5, 2.0, and 2.5), and in free flight.

The ground effect is simulated by PAN AIR and AVL through the use of the method of images. The ground plane is represented by an image of the paneled geometry, placed and oriented as though it is reflected on the surface of the ground, so that, the ground plane does not have normal velocity, only tangential component.

The semi-span wing simulated by PAN AIR is divided in 979 panels, and consists of five surface networks defined as indirect condition on impermeable thick surface and four wake surface networks. The wing simulated by AVL has a vortex lattice with 920 control points, twenty lattice chordwise divisions and forty-six spanwise divisions in a cosine distribution. The Mach number was equal to 0.15 for both simulations.

A. Simulation the Flowfield around the ANCE

The potential flowfield around the airplane was simulated by PAN AIR and AVL codes. Using both codes, the airplane was modeled excluding the landing gear and the camera due to the fact that the contribution of these components to inviscid forces and moments is assumed negligible.

The geometry simulated in PAN AIR consists of 3,009 panels, representing the half of the airplane. The geometry was divided in twenty-seven surface networks; twenty-two are defined as indirect condition on impermeable thick surface (for lifting surfaces). Four are defined as direct condition on impermeable thick surface (for non-lifting surface), and one as base surface condition. There are seventeen wake surface networks to perform the Kutta condition (zero vorticity at trailing-edges and body bases). Figure 2 shows the paneled geometry of the ANCE used in the analysis. Flow symmetry condition is imposed to obtain the complete flowfield, pressure and forces. The method of images is used to simulate the ground plane. Lift, induced drag and pitching moment coefficients were obtained at 0.15 Mach number, at different angles of attack and different non-dimensional heights above ground.

The stability coefficients were computed by the AVL code. The vortex lattice geometry of the ANCE has 1,160 horseshoe vortices, and this was divided in fourteen surfaces and two bodies. The booms were modeled as slender
bodies and the fuselage was idealized with cruciform shapes;\textsuperscript{30} this body simulation is highly computationally efficient for load distribution.\textsuperscript{31} Figure 3 shows the vortex-lattice geometry of the ANCE.

B. Maximum Lift Coefficient Estimation

The maximum lift coefficients in and out-of-ground effect were estimated by a variation of the classic Valarezo and Chin method.\textsuperscript{32} The Valarezo and Chin method or Pressure Difference Rule is a stripwise analysis of an inviscid flowfield to determine an estimate value of the maximum lift coefficient of the entire wing. It assumes that all of the viscous effects are generally local two-dimensional natures. Experimental data indicates that at the maximum lift condition, there is a pressure difference between the suction peak of the airfoil and its trailing-edge at a given combination of Reynolds number and Mach number. Consequently, at a given flow condition, there is a pressure difference that indicates when maximum lift is attained.\textsuperscript{33} Although this method is based on two-dimensional data, it may be used for lifting three–dimensional configurations and it has been validated for those cases.\textsuperscript{32,33}

In this paper, the three–dimensional inviscid flowfield at different non-dimensional heights above ground were computed by PANAIR,\textsuperscript{20} and the pressure difference at the maximum lift coefficient for the two-dimensional sections along the span were obtained by a two-dimensional panel method coupled with a boundary-layer. This is XFOIL version 6.94.\textsuperscript{34} The original Valarezo and Chin method uses a large set of experimental data to generate a generic set of curves that shows the pressure difference as a function of both Mach number and Reynolds number. The variation of the Valarezo and Chin method used here was developed by Syms,\textsuperscript{8} who estimated \( C_{L_{\text{max}}} \) of a Fokker F28 Mk1000 in free flight and in ground effect and validated these against wind tunnel data.

XFOIL is an open-source program created for the design and analysis of isolated airfoils. The code uses a simple linear-vorticity stream function panel method for the inviscid formulation, and it incorporates a Karman–Tsien compressibility correction. The boundary-layer free transition occurs when an \( e^\alpha \) criterion is achieved.\textsuperscript{34} The different wing airfoil sections were simulated at a Reynolds number equal to 1.413\( \times 10^6 \) and Mach 0.15 at steady, incompressible, and viscous flow with free transition criteria at \( n=9 \) and 200 panels around the section. No roughness effects were considered on the surface. The lift curve was obtained and the pressure coefficient difference at maximum lift coefficient was extracted.

IV. Results and Discussion

A. Simulation of a single wing

Figure 4 shows the drag reduction factor as a function of non-dimensional height estimated by analytical Eqs. (1) and (2), and numerical methods. It could be observed that for values of \( z_e \) larger than 0.5, the data achieved by numerical simulation are in excellent agreement with analytical results. The data obtained by the vortex lattice method is quite different about analytical data for \( z_e \) less than 0.25. A difference of 39.9% and 28.8% is achieved at \( z_e=0.1 \) between AVL data and analytical results from Eq. (1) and Eq. (2), respectively. At this same height, data computed by PAN AIR has a difference of 1.91% and 9.66% with respect to Eq. (1) and Eq. (2), respectively.

B. The Complete Airplane

Figures 5-7 and Table 1 show the data obtained by PAN AIR and AVL in free flight. The PAN AIR paneled representation of the ANCE was previously used to model the ANCE without wing twist,\textsuperscript{14} and the lift and drag coefficients were in between excellent and good agreement with the experimental data present in Ref. 13.

The induced drag factor, the lift coefficient at zero angle of attack and the pitching moment slope obtained with the Athena Vortex Lattice are in excellent correlation with data achieved by PAN AIR. A difference of 12.61% is observed between the lift slope predicted by the panel method code and that one computed using AVL. The value of pitching moment coefficient at zero angle of attack computed by AVL does not have correlation with PAN AIR data.
Tables 2 and 3 show the aerodynamic characteristics and stability coefficients achieved by the Athena Vortex Lattice and PAN AIR, respectively. Figures 8-15 show the variations of the stability coefficients and other characteristics as a function of the non-dimensional height above ground respect to data computed by PAN AIR (Fig. 8) and AVL (Figs. 9-15) in free flight. Numerical anomalies were encountered in a calculation attempted with $z_e$ less than 0.577 using AVL. It was felt that this problem was probably caused by the close proximity of the actual and image model vortex wakes.

The values computed herein demonstrate that the ground effect has a strong influence on the longitudinal stability coefficients. When the airplane approaches the ground $C_{L_e}$ and $C_{L_\delta e}$ increase, $C_{M_{\alpha e}}$, $C_{M_{\delta e}}$, and $C_{D_e}$ reduce, while $C_{L_\alpha}$ and $C_{M_{\alpha}}$ do not experiment changes. The pitching moment coefficient is less than zero at each height above ground and the location of the stick fixed neutral point ($h_\alpha$) increases in ground effect. The values of static height stability ($H_S$) computed by AVL and PAN AIR are positive and they decrease with ground proximity. This means that the airplane nose-down pitching motion exceeds the stabilizing influence of $C_{L_\alpha}$, and this must be more unstable when it approaches the ground. However, the value of the static height stability calculated by PAN AIR at $z_e=1.077$ is negative, indicating that the airplane is statically stable at this height.

The lateral-directional stability coefficients present a few variations with ground effect. When the airplane reaches a $z_e=0.577$, $C_{L_\beta}$ and $C_{\delta_\beta e}$ decrease slightly, $C_{n_\beta}$ reduces 344.4%, and $C_{\beta_\beta}$ augments 16.6%. $C_{\delta e}$ increases 25% when the non-dimensional height above ground is equal to 1.077, and 107.5% when $z_e=0.577$. The values of $C_{L_\beta}$ and $C_{\alpha_\theta}$ are independent of $z_e$ and criterions for the lateral and directional static stability are satisfied ($C_{L_\beta}<0$, $C_{\alpha_\theta}>0$).
It is observed from the data obtained by PAN AIR and AVL that the value of the induced drag factor diminishes when the airplane approaches the ground plane. If the minimum drag coefficient stays constant, the maximum lift-drag ratio increases. The value of induced drag factor at \(z_e=0.074\) (landing gear in contact with the ground) computed by PAN AIR increases 11.4\% regarding free flight. This may be incorrect, because in extreme ground

| Table 2. Aerodynamic characteristics and stability coefficients computed by the Athena Vortex Lattice given in 1/rad. |
|---|---|---|---|---|---|---|
| \(z_e\) | 0.577 | 1.077 | 1.577 | 2.077 | 2.577 | \(\infty\) |
| \(k\) | 0.048 | 0.0495 | 0.0500 | 0.0503 | 0.0504 | 0.0506 |
| \(h_o\) | 0.836c | 0.829c | 0.827c | 0.826c | 0.827c | 0.825c |
| \(C_{Lo}\) | 0.319 | 0.314 | 0.312 | 0.312 | 0.312 | 0.312 |
| \(C_{Mo}\) | -0.108 | -0.103 | -0.101 | -0.101 | -0.101 | -0.100 |
| \(C_{Do}\) | 0.234 | 0.240 | 0.242 | 0.243 | 0.243 | 0.244 |
| \(C_{Lo}\) | 4.927 | 4.853 | 4.830 | 4.824 | 4.819 | 4.813 |
| \(C_{Ltc}\) | 0.682 | 0.659 | 0.653 | 0.653 | 0.653 | 0.653 |
| \(C_{Ma}\) | -2.888 | -2.807 | -2.785 | -2.779 | -2.779 | -2.773 |
| \(C_{M/e}\) | -2.756 | -2.744 | -2.739 | -2.739 | -2.739 | -2.739 |
| \(C_{Lc}\) | -1.1\times10^{-2} | -3.0\times10^{-3} | -1.0\times10^{-3} | -4.0\times10^{-4} | -3.0\times10^{-5} | 0 |
| \(C_{Lw}\) | 1.4\times10^{-3} | 6.0\times10^{-4} | -2.0\times10^{-4} | -1.2\times10^{-3} | -1.0\times10^{-4} | 0 |
| \(C_{Le}\) | 1.1\times10^{-2} | 2.8\times10^{-3} | 8.0\times10^{-4} | 4.0\times10^{-4} | 2.0\times10^{-5} | 0 |
| \(HS\) | 8.3\times10^{-3} | 1.8\times10^{-3} | 3.9\times10^{-4} | 2.9\times10^{-4} | 4.7\times10^{-6} | 0 |
| \(C_{Yp}\) | -0.533 | -0.527 | -0.527 | -0.527 | -0.527 | -0.527 |
| \(C_{Yp}\) | -0.045 | -0.055 | -0.055 | -0.055 | -0.055 | -0.055 |
| \(C_{Yr}\) | 0.455 | 0.455 | 0.455 | 0.455 | 0.455 | 0.455 |
| \(C_{Yr}\) | -0.332 | -0.332 | -0.332 | -0.332 | -0.332 | -0.332 |
| \(C_{Yr}\) | 1.72\times10^{-4} | -0.0017 | -0.0023 | -0.0023 | -0.0023 | -0.0023 |
| \(C_{Yr}\) | -0.057 | -0.057 | -0.057 | -0.057 | -0.057 | -0.057 |
| \(C_{Yr}\) | -0.546 | -0.546 | -0.546 | -0.546 | -0.546 | -0.546 |
| \(C_{Yr}\) | 0.091 | 0.091 | 0.091 | 0.091 | 0.091 | 0.091 |
| \(C_{Yr}\) | -0.023 | -0.023 | -0.023 | -0.023 | -0.023 | -0.023 |
| \(C_{Yr}\) | -0.327 | -0.321 | -0.321 | -0.321 | -0.321 | -0.321 |
| \(C_{Yr}\) | 0.132 | 0.132 | 0.132 | 0.132 | 0.132 | 0.132 |
| \(C_{Yr}\) | -0.004 | -0.001 | -0.001 | -0.001 | -0.001 | -0.001 |
| \(C_{Yr}\) | -0.182 | -0.182 | -0.182 | -0.182 | -0.182 | -0.182 |
| \(C_{Yr}\) | 0.172 | 0.172 | 0.172 | 0.172 | 0.172 | 0.172 |
| \(C_{Yr}\) | 0.011 | 0.011 | 0.011 | 0.011 | 0.011 | 0.011 |

| Table 3. Aerodynamic data computed by PAN AIR given in 1/rad. |
|---|---|---|---|---|---|---|
| \(z_e\) | 0.074 | 0.577 | 1.077 | 1.577 | 2.077 | 2.577 | \(\infty\) |
| \(k\) | 0.055 | 0.028 | 0.040 | 0.046 | 0.046 | 0.048 | 0.049 | 0.049 |
| \(h_o\) | 0.981c | 0.916c | 0.900c | 0.900c | 0.898c | 0.897c | 0.893c |
| \(C_{Lo}\) | 0.321 | 0.311 | 0.309 | 0.309 | 0.309 | 0.309 | 0.309 |
| \(C_{Mo}\) | -0.071 | -0.022 | -0.012 | -0.012 | -0.011 | -0.011 | -0.009 |
| \(C_{Do}\) | 4.366 | 4.274 | 4.269 | 4.269 | 4.269 | 4.269 | 4.274 |
| \(C_{Lq}\) | -0.0186 | -0.004 | -0.0008 | 0.0008 | -0.0002 | -6\times10^{-7} | 0 |
| \(C_{Le}\) | 0.0974 | 0.0196 | 0 | 0.0018 | 0.0012 | 3\times10^{-6} | 0 |
| \(HS\) | 0.11465 | 0.02542 | -0.0008 | 0.00357 | 0.00165 | 4\times10^{-6} | 0 |
| \(C_{Lmax}\) | 1.551 | 1.592 | 1.600 | 1.604 | 1.611 | 1.612 | 1.613 |

It is observed from the data obtained by PAN AIR and AVL that the value of the induced drag factor diminishes when the airplane approaches the ground plane. If the minimum drag coefficient stays constant, the maximum lift-drag ratio increases. The value of induced drag factor at \(z_e=0.074\) (landing gear in contact with the ground) computed by PAN AIR increases 11.4\% regarding free flight. This may be incorrect, because in extreme ground flight conditions, the induced drag factor may not diminish as expected.
The maximum lift coefficients at each non-dimensional height were estimated using the modified Valarezo and Chin method previously mentioned. The maximum lift coefficient reduces with ground proximity. The reduction of $C_{L_{\text{max}}}$ is 3.8% when the airplane landing gear is in contact with the ground. This does not represent a dangerous condition, but it is necessary it be taken into account to estimate the performance of the aircraft during landing and take-off.

C. Dynamic Stability and Response in Ground Effect and in free flight

The longitudinal and lateral-directional equations of motions were written for each value of non-dimensional height above ground using the data calculated by the vortex lattice method. The minimum drag coefficient was previously estimated via wind tunnel test in Ref. 13 and is equal to 0.0266. The rolling, pitching and yawing moments of inertia used are 150, 400 and 400 kg·m², respectively. The variations of lift and pitching moment coefficients with rate of change of angle of attack are 2.3342 and −10.0497 rad⁻¹, respectively and were estimated by...
Both values were independent of height above ground. No variations in air density are assumed to estimate the stability derivatives, in order to evaluate only the influence of ground effect in dynamic response.

Then, the eigenvalues from matrices $A$ and $C$ were extracted at each $z_e$.

Table 4 shows the eigenvalues calculated for each mode of response. The solution of the longitudinal system of equations generates five eigenvalues: two complex pairs representing the short-period and the phugoid mode, and the other root, real, representing a non-oscillatory mode. This root represents the non-dimensional height mode. The short period and the phugoid mode roots are complex and have negative real parts at each value of $z_e$, indicating that both modes are damped oscillatory and convergent motions. The roots for non-dimensional height mode equal to or less than 2.5 are real and positive, indicating a divergent mode motion. In free flight, the value of this root tends to zero and does not describe any motion.

The eigenvalues obtained from matrix $C$ are associated with linearized lateral-directional motion, and there are four. A pair of complex roots represents the Dutch roll mode; the negative real part indicates that it is damped oscillatory motion for all the non-dimensional heights above ground. The root for roll motion is real and negative.
indicating a stable and heavily damped motion. The real and positive root is associated with divergent and non-oscillatory spiral mode. Although the spiral and the roll mode are not affected by the ground effect and the short-period and the Dutch roll mode are poorly influenced by this phenomenon, the phugoid mode was found to be quite sensitive to \( z_e \) variations.

To demonstrate the effect of the \( z \) derivatives on the dynamic response, the matrix \( A \) was re-written, assuming that the values of \( z \) derivatives are equal to zero. Table 4 shows the real and imaginary part of the eigenvalues for the short-period and the phugoid when the \( z \) derivatives are neglected. It can be appreciated that the short-period and the phugoid modes are slightly influenced by ground effect in this specific case.

The eigenvalues obtained were used to determine the characteristics of the six modes of motion. Table 5 contains the calculated values for these characteristics, including those ones when the \( z \) derivatives are assumed equal to zero. When the \( z \) derivatives are included, it can be observed that the two modes of response mainly affected by the proximity of the ground are the phugoid and the non-dimensional height. It can be appreciated that the time to double amplitude of the non-dimensional height mode augments when the aircraft height above ground increases, while in free flight, \( t_2 \) becomes infinite. Any of these phenomena cannot be appreciated when the \( z \) derivatives are neglected.

Table 4. Eigenvalues for different modes of response.

<table>
<thead>
<tr>
<th>( z_e )</th>
<th>0.577</th>
<th>1.077</th>
<th>1.577</th>
<th>2.077</th>
<th>2.577</th>
<th>( \infty )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Including the ( z ) derivatives</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Phugoid Real</td>
<td>-0.0065</td>
<td>-0.0055</td>
<td>-0.0044</td>
<td>-0.0038</td>
<td>-0.0043</td>
<td>-0.0043</td>
</tr>
<tr>
<td>Phugoid Imaginary</td>
<td>±0.2389</td>
<td>±0.2328</td>
<td>±0.2316</td>
<td>±0.2318</td>
<td>±0.2311</td>
<td>±0.2313</td>
</tr>
<tr>
<td>Short period Real</td>
<td>-1.5294</td>
<td>-1.4845</td>
<td>-1.4723</td>
<td>-1.4706</td>
<td>-1.4681</td>
<td>-1.4684</td>
</tr>
<tr>
<td>Short period Imaginary</td>
<td>±3.1021</td>
<td>±3.0828</td>
<td>±3.0767</td>
<td>±3.0778</td>
<td>±3.0793</td>
<td>±3.0792</td>
</tr>
<tr>
<td>Non-dimensional height Real</td>
<td>0.1312</td>
<td>0.0362</td>
<td>0.0101</td>
<td>0.0035</td>
<td>0.0001</td>
<td>0</td>
</tr>
<tr>
<td>Not including the ( z ) derivatives</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Phugoid Real</td>
<td>-0.0041</td>
<td>-0.0042</td>
<td>-0.0043</td>
<td>-0.0043</td>
<td>-0.0043</td>
<td>-0.0043</td>
</tr>
<tr>
<td>Phugoid Imaginary</td>
<td>±0.2341</td>
<td>±0.2319</td>
<td>±0.2313</td>
<td>±0.2311</td>
<td>±0.2311</td>
<td>±0.2313</td>
</tr>
<tr>
<td>Short period Real</td>
<td>-1.4661</td>
<td>-1.4677</td>
<td>-1.4674</td>
<td>-1.4683</td>
<td>-1.468</td>
<td>-1.4684</td>
</tr>
<tr>
<td>Short period Imaginary</td>
<td>±3.093</td>
<td>±3.0811</td>
<td>±3.0765</td>
<td>±3.0776</td>
<td>±3.0793</td>
<td>±3.0792</td>
</tr>
</tbody>
</table>

Based on the differences between the roots and dynamic characteristics estimated including the \( z \) derivatives, and those ones computed assuming these derivatives equal to zero, an analysis was made to determine the free longitudinal response of the airplane for a given disturbance. The classical four-order Runge-Kutta method was used to obtain the free response of motion as a function of time. A sinusoidal elevator variation of 5 deg for 0.5 s was the initial disturbance at each simulation.

Figure 16 shows the time history of the characteristics for the initial disturbance for non-dimensional heights equal to 0.577, 1.077, 1.577, and in free flight, including the \( z \) derivatives. It is observed that the variables associated with the phugoid mode, \( \Delta u \) and \( \Delta \theta \), are strongly affected when the distance between the airplane and the ground plane is reduced. The variables associated with the short-period mode are slightly affected by the proximity to the ground and decay to zero within 3 to 4 s. However, the divergent non-dimensional height mode response disturbs the other variables and all of them tend to be divergent.

Figures 17-19 show the time history obtained at \( z_e \) equal to 0.577, 1.077 and 1.577, respectively, when the \( z \) derivatives are neglected in comparison to those ones obtained previously (including the \( z \) derivatives). The difference between the time history obtained including and not including the \( z \) derivatives is more evident at \( z_e=0.577 \). At \( z_e \) equal to 1.077 and 1.577, this is more perceptible in \( \Delta \theta \), which is a variable associated with the phugoid mode. If the \( z \) derivatives are neglected, the time response is convergent, and the airplane is dynamically stable in ground effect, which is incorrect because the values of \( HS \) estimated by the data obtained using AVL are greater than zero, and the airplane is expected to be unstable in ground proximity.
Rohlf and Friehmelt\textsuperscript{37} presented the flight test time history of the X-31 combat aircraft in ground effect against predicted data with and without ground effect. The time history data predicted with ground effect in touch and go maneuvers agree very well with experimental data, but the result without ground effect had deficiencies like the data presented in the present paper that exclude the $z$ derivatives.

D. Flying Qualities

The dynamic stability characteristics are used to define the flying qualities of the ANCE in ground effect and in free flight with reference to the Cooper and Harper scale\textsuperscript{38} The specifications of flying qualities requirements for airplanes are presented by Hodgkinson,\textsuperscript{39} and for remotely piloted vehicles can be found in the Prosser and Wiler report.\textsuperscript{40} The ANCE is classified according to size and maneuverability as class III, in flight phases B and C. The category C refers to terminal and initial flight phases, which means landing and take-off. A summary of flying qualities requirements is presented in Appendix B.

\begin{table}[h]
\centering
\caption{Longitudinal and Lateral-directional dynamic characteristics.}
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline
 & \multicolumn{7}{|c|}{Including the $z$ derivatives} \\
\hline
 & $z_e$ & 0.577 & 1.077 & 1.577 & 2.077 & 2.577 & \infty \\
\hline
Phugoid & $t_{1/2}$, s & 106.1538 & 125.4545 & 156.8182 & 181.5789 & 160.4651 & 160.4651 \\
 & $T$, s & 26.3005 & 26.9896 & 27.1295 & 27.1061 & 27.1882 & 27.1647 \\
 & $\zeta$ & 0.0272 & 0.0236 & 0.0190 & 0.0164 & 0.0186 & 0.0186 \\
 & $\omega_n$, rad/s & 0.2390 & 0.2329 & 0.2316 & 0.2318 & 0.2311 & 0.2313 \\
 & $N_{1/2}$, cycles & 4.0362 & 4.6484 & 5.7804 & 6.9888 & 5.9020 & 5.9071 \\
\hline
Short period & $t_{1/2}$, s & 0.4512 & 0.4648 & 0.4687 & 0.4692 & 0.4700 & 0.4699 \\
 & $T$, s & 2.0255 & 2.0381 & 2.0422 & 2.0415 & 2.0405 & 2.0405 \\
 & $\zeta$ & 0.4422 & 0.4339 & 0.4317 & 0.4311 & 0.4304 & 0.4304 \\
 & $\omega_n$, rad/s & 3.4586 & 3.4216 & 3.4108 & 3.4111 & 3.4114 & 3.4114 \\
 & $N_{1/2}$, cycles & 0.2227 & 0.2281 & 0.2295 & 0.2298 & 0.2303 & 0.2303 \\
 & $CAP$ & 1.8369 & 1.7880 & 1.7758 & 1.7732 & 1.7739 & 1.7725 \\
\hline
Non-dimensional height & $t_2$, s & 5.2591 & 19.0608 & 68.3168 & 197.1429 & 6900.0000 & \infty \\
\hline
Dutch Roll & $t_{1/2}$, s & 2.7901 & 2.7567 & 2.7468 & 2.7414 & 2.7392 & 2.7359 \\
 & $\zeta$ & 0.1190 & 0.1194 & 0.1195 & 0.1196 & 0.1196 & 0.1197 \\
 & $\omega_n$, rad/s & 2.0786 & 2.0969 & 2.1021 & 2.1048 & 2.1058 & 2.1076 \\
 & $T$, s & 3.0228 & 2.9964 & 2.9891 & 2.9852 & 2.9837 & 2.9811 \\
\hline
Roll & $t_{1/2}$, s & 0.2087 & 0.2067 & 0.2062 & 0.2059 & 0.2058 & 0.2056 \\
 & $\sigma^{-1}$, s & 0.0219 & 0.0214 & 0.0213 & 0.0213 & 0.0213 & 0.0212 \\
\hline
Spiral & $t_2$, s & 130.1887 & 130.1887 & 130.1887 & 130.1887 & 130.1887 & 130.1887 \\
\hline
Not including the $z$ derivatives & & & & & & & \\
\hline
Phugoid & $t_{1/2}$, s & 168.2927 & 164.2857 & 160.4651 & 160.4651 & 160.4651 & 160.4651 \\
 & $T$, s & 26.8397 & 27.0944 & 27.1647 & 27.1882 & 27.1882 & 27.1647 \\
 & $\zeta$ & 0.0175 & 0.0181 & 0.0186 & 0.0186 & 0.0186 & 0.0186 \\
 & $\omega_n$, rad/s & 0.2341 & 0.2319 & 0.2313 & 0.2311 & 0.2311 & 0.2313 \\
 & $N_{1/2}$, cycles & 6.2703 & 6.0635 & 5.9071 & 5.9020 & 5.9020 & 5.9071 \\
\hline
Short period & $t_{1/2}$, s & 0.4706 & 0.4701 & 0.4702 & 0.4699 & 0.4700 & 0.4699 \\
 & $T$, s & 2.0314 & 2.0393 & 2.0423 & 2.0416 & 2.0405 & 2.0405 \\
 & $\zeta$ & 0.4283 & 0.4301 & 0.4305 & 0.4306 & 0.4303 & 0.4304 \\
 & $\omega_n$, rad/s & 3.4229 & 3.4128 & 3.4085 & 3.4099 & 3.4113 & 3.4114 \\
 & $N_{1/2}$, cycles & 0.2317 & 0.2305 & 0.2302 & 0.2302 & 0.2304 & 0.2303 \\
 & $CAP$ & 1.7991 & 1.7788 & 1.7734 & 1.7720 & 1.7738 & 1.7725 \\
\hline
\end{tabular}
\end{table}
Figure 16. Longitudinal response of the airplane at different heights above ground.

Figure 17. Longitudinal response of the airplane at $z_e=0.577$.

Figure 18. Longitudinal response of the airplane at $z_e=1.077$.

Figure 19. Longitudinal response of the airplane at $z_e=1.577$. 
The data presented in table 5 show that the airplane satisfies the requirements to achieve level 1 in categories B and C for the short period mode, and level 2 for the phugoid at different heights above ground. Figure 20 shows the rating in free flight and at $z_e=0.577$ for category C. These demonstrate that the short-period qualities are slightly affected by ground effect. Level 1 is achieved for roll, spiral and Dutch roll modes at each non-dimensional height.

No requirements are presented in the literature for the non-dimensional height mode. However, the non-dimensional height mode motion is a divergent mode like the spiral mode. For this reason, the spiral mode requirements are used herein to qualify the non-dimensional height mode. As a result, the time to double the amplitude of non-dimensional height mode is lower than the requirements for time to double the amplitude achieving level 3 at $z_e=0.577$, level 2 at $z_e=1.077$, and level 1 at $z_e>1.077$.

V. Conclusions

The present paper demonstrates that the performance and the stability of the ANCE are considerably influenced by the proximity to the ground.

Panel method and vortex lattice method codes were used to compute the entire flowfield around a wing and the ANCE in free flight and in ground effect. Based on these experiences, the panel method code can simulate the effect of ground proximity on lift and on induced drag very well. The vortex lattice method does not reach a very good correlation at $z_e$ less than 0.25.

The stability coefficients of the ANCE were estimated at different heights above ground using the vortex lattice method, and the equations of motion were written. It could be concluded that although the longitudinal static stability criterion is satisfied ($C_{Ma}<0$), the static height stability must be accomplished to ensure that an airplane is statically stable in ground effect. Additionally, the longitudinal static and dynamic stability of the Unmanned Airplane for Ecological Conservation is strongly influenced by ground effect. The phugoid and the non-dimensional height are the most affected modes of motion in ground effect. The lateral-directional stability is slightly influenced by this phenomenon. Only the flying qualities related with non-dimensional height mode are considerably influenced by ground effect. The airplane achieved level 1 in free flight, level 2 at $z_e=1.077$, and level 3 at $z_e=0.577$.

It is important to emphasize that the $z$ derivatives affect considerably the longitudinal dynamic response in ground effect. Therefore, it would be a mistake to take into account only the stability derivatives at a given height to study the dynamic response of an aircraft in ground effect neglecting the $z$ derivatives.

Appendix A

Tables A1 and A2 show the expressions of the stability derivatives based on the stability coefficients.
### Table A1. Summary of longitudinal derivatives.

<table>
<thead>
<tr>
<th>Derivative</th>
<th>S.I. Unit</th>
<th>Derivative</th>
<th>S.I. Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X_u = -(C_{Da} + 2C_{Do}) \frac{Q \cdot S}{m \cdot u_o} )</td>
<td>s(^{-1})</td>
<td>( Z_{\delta e} = -C_{L\delta e} \frac{Q \cdot S}{m} )</td>
<td>m/s(^2)</td>
</tr>
<tr>
<td>( X_w = -(C_{Da} - C_{Lo}) \frac{Q \cdot S}{m \cdot u_o} )</td>
<td>s(^{-1})</td>
<td>( M_q = C_{Mq} \frac{c \cdot Q \cdot S \cdot c}{2u_o} )</td>
<td>s(^{-1})</td>
</tr>
<tr>
<td>( X_z = -C_{Dz} \frac{Q \cdot S}{m \cdot u_o} )</td>
<td>s(^{-1})</td>
<td>( M_u = C_{Mu} \frac{Q \cdot S \cdot c}{u_o I_y} )</td>
<td>(m/s(^1))</td>
</tr>
<tr>
<td>( X_{\delta e} = -C_{D\delta e} \frac{Q \cdot S}{m} )</td>
<td>m/s(^2)</td>
<td>( M_w = C_{M\delta e} \frac{Q \cdot S \cdot c}{u_o I_y} )</td>
<td>(m/s(^1))</td>
</tr>
</tbody>
</table>

### Table A2. Summary of lateral-directional derivatives.

<table>
<thead>
<tr>
<th>Derivative</th>
<th>S.I. Unit</th>
<th>Derivative</th>
<th>S.I. Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y_\beta = C_{y\beta} \frac{Q \cdot S}{m} )</td>
<td>m/s(^2)</td>
<td>( N_{\delta e} = C_{u\delta e} \frac{Q \cdot S \cdot b}{I_z} )</td>
<td>s(^{-2})</td>
</tr>
<tr>
<td>( Y_\rho = C_{y\rho} \frac{b \cdot Q \cdot S}{2u_o} )</td>
<td>m/s</td>
<td>( N_{\delta} = C_{u\delta} \frac{Q \cdot S \cdot b}{I_z} )</td>
<td>s(^{-2})</td>
</tr>
<tr>
<td>( Y_r = C_{y\rho} \frac{b \cdot Q \cdot S}{2u_o} )</td>
<td>m/s</td>
<td>( L_{\beta} = C_{\beta} \frac{b^2 \cdot Q \cdot S}{2u_o} )</td>
<td>s(^{-2})</td>
</tr>
<tr>
<td>( Y_{\delta e} = C_{\delta e} \frac{Q \cdot S}{m} )</td>
<td>m/s(^2)</td>
<td>( L_{\beta} = C_{\beta} \frac{b^2 \cdot Q \cdot S}{2u_o} )</td>
<td>s(^{-1})</td>
</tr>
<tr>
<td>( Y_{\delta} = C_{\delta e} \frac{Q \cdot S}{m} )</td>
<td>m/s(^2)</td>
<td>( L_r = C_r \frac{b^2 \cdot Q \cdot S}{2u_o} )</td>
<td>s(^{-1})</td>
</tr>
<tr>
<td>( N_{\beta} = C_{\delta e} \frac{Q \cdot S \cdot b}{I_z} )</td>
<td>s(^{-2})</td>
<td>( L_{\delta e} = C_{\delta e} \frac{Q \cdot S \cdot b}{I_z} )</td>
<td>s(^{-2})</td>
</tr>
<tr>
<td>( N_{\beta} = C_{\delta e} \frac{b^2 \cdot Q \cdot S}{2u_o} )</td>
<td>s(^{-1})</td>
<td>( L_{\delta e} = C_{\delta e} \frac{Q \cdot S \cdot b}{I_z} )</td>
<td>s(^{-2})</td>
</tr>
<tr>
<td>( N_r = C_{\delta e} \frac{b^2 \cdot Q \cdot S}{2u_o} )</td>
<td>s(^{-1})</td>
<td>( L_{\delta e} = C_{\delta e} \frac{Q \cdot S \cdot b}{I_z} )</td>
<td>s(^{-2})</td>
</tr>
</tbody>
</table>

### Appendix B

Tables B1 – B4 show the flying qualities requirements for class III aircraft, in categories B and C. 39,40

#### Table B1. Short period mode damping ratio requirements.

<table>
<thead>
<tr>
<th>Level</th>
<th>Category C</th>
<th>Category B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Minimum, ( \zeta )</td>
<td>Maximum, ( \zeta )</td>
</tr>
<tr>
<td>1</td>
<td>0.35</td>
<td>1.3</td>
</tr>
<tr>
<td>2</td>
<td>0.25</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>0.15</td>
<td>–</td>
</tr>
</tbody>
</table>
Table B2. The phugoid mode requirements.

<table>
<thead>
<tr>
<th>Level</th>
<th>( \zeta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \zeta &gt; 0.4 )</td>
</tr>
<tr>
<td>2</td>
<td>( \zeta &gt; 0 )</td>
</tr>
<tr>
<td>3</td>
<td>( t_{1/2} &gt; 55 \text{s} )</td>
</tr>
</tbody>
</table>

Table B3. Spiral and roll modes requirements for aircraft class III, categories B and C.

<table>
<thead>
<tr>
<th>Level</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum roll ( \sigma ), s</td>
<td>1.4</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>Minimum spiral ( t_{2} ), s</td>
<td>20</td>
<td>12</td>
<td>4</td>
</tr>
</tbody>
</table>

Table B4. The Dutch roll mode requirements for aircraft class III, categories B and C.

<table>
<thead>
<tr>
<th>Level</th>
<th>Category</th>
<th>Minimum, ( \zeta )</th>
<th>Minimum, ( (\zeta \omega_{n}) ), rad/s</th>
<th>Minimum, ( \omega_{n} ), rad/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>B</td>
<td>0.08</td>
<td>0.15</td>
<td>0.4</td>
</tr>
<tr>
<td>2</td>
<td>C</td>
<td>0.08</td>
<td>0.1</td>
<td>0.4</td>
</tr>
<tr>
<td>3</td>
<td>All</td>
<td>0.02</td>
<td>0.05</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Acknowledgments

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References


