EFFECT OF NEARBY LIGHTNING ON OVERHEAD ELECTRIC LINES:
A COMPARISON OF SIMULATION METHODS

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Abstract - A survey of the methods proposed in the literature to evaluate the effect of nearby lightning strokes on electric power transmission / distribution circuits is performed, describing the assumptions taken as well as the mathematical tools applied at each of the simulation stages of the physical phenomena. A comparison is made between the approaches proposed by Master & Uman [1] and Chowdhury [2] to describe the electromagnetic field associated to the atmospheric discharge. After validation, both methodologies are applied to the same reference problem. Results are presented and analyzed, reflecting interesting differences in the figures obtained by each of the approaches for the outage rates of the electric circuits presented in [3], chosen as test cases, with a tendency of the method of Chowdhury to estimate higher overvoltages.

I. INTRODUCTION

The importance of the effect of the electromagnetic field irradiated by lighting strokes that takes place on the vicinity of overhead electric circuits has been demonstrated, and is presently subject of study and research.

Some authors state that indirect lightning strokes, due to its high frequency of occurrence, constitute a major cause of the failure of electric power distribution circuits, surpassing the effect of direct strokes in the performance of the circuits. [4,5,6]

The physical mechanisms of overvoltage induction in electric circuits have been widely studied and analyzed; however, as different approaches have been proposed and, considering the lack of field tests and measurements, the verification of the relative goodness of each of the approaches remains as an interesting point to be verified.

Four different sequential stages may be distinguished in the calculations of the induced overvoltages due to lightning activity in the vicinity of an electric circuit, as follows:

A.- Modeling of the electric discharge.
B.- Simulation of the electromagnetic field produced by the lightning discharge.
C.- Determination of the induced voltages on the conductors of the electric circuits.
D.- Simulation of the traveling wave phenomena on the conductors of the electric circuit.

Once the electromagnetic wave on the circuit has been simulated, the effect of the phenomena on the behaviour of the insulation of the electric circuit can be estimated by calculating the probability of flashover of the insulation of the circuit. Monte Carlo techniques can be applied to obtain the behaviour of the circuit for the generality of the cases.

II. INDUCED VOLTAGES DUE TO NEARBY LIGHTNING

The main four sequential stages or steps that integrate the simulation process of the physical phenomena that takes place after a lightning stroke in the vicinity of an electric circuit are described in this section.

A.- Modeling of the electric discharge.

Bewley [7] based his analysis in the collapse of the electrostatic field in the neighborhood of the circuit caused by the discharge of the cloud, ignoring the effect of the electromagnetic fields associated to the return stroke. Wagner and McCann [8] showed that the impact of the fields associated to the return stroke is more important than the effect of the electrostatic fields of the cloud on the induced overvoltage phenomena on nearby electric circuits.

The speed of the discharge is a very important variable in the modeling of the atmospheric discharge, affecting the results significantly. [9,10] Berger [11] suggested a value within 20 and 110 m/µs. Bruce and Golde, [12] stated that the speed of the return stroke discharge reduces exponentially as the discharge travels form the ground to the cloud:

\[ v(t) = v_0 \exp(-\gamma t) \quad [\text{m/seg}] \]  

Lundholm and Rusck established a relation between the speed and the magnitude of the discharge: [9,10]

\[ v = \frac{c}{\sqrt{1 + \frac{500}{I}}} \quad [\text{m/µseg}] \]  

where:
- \( c = \) speed of light
- \( I = \) magnitude of the current of the return stroke discharge

A constant value of 0.3 times the speed of light was assumed for the speed of the return stroke, during the calculations performed in this work.
In general, the plasmatic channel of the atmospheric discharge is considered as an one dimension vertical antenna, over the ground plane, with an electric current distribution in space and time along its length, described by a mathematical model.

The models are stated in terms of the expected value of the current at ground level, due to the physical limitations associated to the measurement of the current.

Different mathematical models have been proposed for the space-time current distribution through the discharge channel, such as the Traveling Current Source, [6] the Modified Transmission Line, [6] and the Uniform Charge Distribution models: [2,13,14,15]

In this work, the Traveling Current Source Model was used for the calculations using the Master & Uman approach, while the Uniform Charge Distribution Model was applied for the simulations using the Chowdhury approach, as suggested by the author.

i.- Traveling Current Source Model: [6]

This model assumes an equivalent current source ascending by the plasmatic channel, with the speed of the return stroke (between 20 and 110 m/µs), liberating an electric current that flows from the traveling source to ground, with the speed of light.

\[
I(z',t) = I(0,t+z'/c) \quad \text{for} \quad z' \leq vt \quad (3.a)
\]

\[
I(z',t) = 0 \quad \text{for} \quad z' > vt \quad (3.b)
\]

where:
- \(I(z',t)\) = electric current in position \(z'\) of the plasmatic channel (kA)
- \(z'\) = position (elevation) in the plasmatic channel (m)
- \(t\) = time (s)
- \(c\) = speed of light (m/s)
- \(v\) = speed of the return stroke

ii.- Modified Transmission Line Model [6]

In this model, the electric current through the plasmatic channel reduces with the elevation from the ground.

\[
I(z',t) = \exp(-z'/\lambda) \cdot I(0,t-z'/v) \quad \text{[kA]} \quad (4)
\]

where:
- \(\lambda\) = decay constant, to take into account the effect of the vertical distribution of the stored charge in the corona shield of the plasmatic channel, drained during the return stroke process (from 1 to 2 km).

iii.- Uniform Charge Distribution Model [13]

In his approach, Chowdhury uses a particular model for the electric discharge. The main premise is that the electric charge is uniformly distributed throughout the plasmatic channel, between the cloud and the ground.

B.- Simulation of the electromagnetic field produced by the lightning discharge.

There are two main tendencies for the calculation of the electromagnetic field generated by the return stroke associated to the lightning stroke:

i) the approach originally proposed by Master & Uman [1], formulated in terms of cylindrical coordinates centered in the axis of the plasmatic channel of the discharge, and

ii) the approach proposed by Chowdhury, [2] formulated in terms of rectangular coordinates.

Both schemes are widely used and referenced in the literature; and although they share some similarities in the assumptions that are made to simplify the problem formulation, each of the two schemes has its own premises.

Presently the research efforts in this area are oriented to the simulation of the effect of a ground with finite conductivity, and of the non verticality of the atmospheric discharge.

i.- Method of Chowdhury [2,13,14,15]

The following premises are the basis for the analysis:

- The electrostatic effect associated to the charge of the cloud, as well as to the building and descent processes of the plasmatic channel of the atmospheric discharge are neglected. Only the induction effect due to the electrostatic and magnetic components of the return stroke discharge are considered in the analysis.

- The discharge channel is assumed to be vertical, with a return stroke wave that starts at the ground plane at \(t = 0\) seg.

- The speed of the return stroke is assumed to be constant, and is kept as a parameter of the formulation.

- The distribution of the electric charge along the discharge channel is supposed to be uniform from the ground to the cloud.

- The conductors of the electric system are considered as ideal conductors, and the conductivity of the ground with an infinite value.

- The analysis is performed assuming a rectangular wave shape for the return stroke. The response for other types of wave shapes are obtained by means of the theorem of Duhamel.

- A rectangular coordinate system is used, defined as shown in Fig.1.
Fig 1. Rectangular Coordinate System Used for the Calculation of the Electromagnetic Fields by Chowdhuri [2]

According to the theory of Chowdhuri, the electromagnetic fields associated to the electric charge and current of the discharge, at any point of space (Xo, Yo, Zo), are given by the relation:

\[
E_{\text{total}} = - (\nabla \phi + \frac{\partial A}{\partial t}) \quad [\text{V/m}] \quad (5)
\]

where:
- \(E_{\text{total}}\) = Total electric field generated by the atmospheric discharge.
- \(\phi\) = Retarded scalar electric potential, associated to the residual stationary charge located in the plasmatic channel.
- \(A\) = Retarded vectorial magnetic potential, associated to the electric current of the return stroke discharge.

It is stated that both components of the total electric field (\(\nabla \phi\) and \(\frac{\partial A}{\partial t}\)), are vertical, and therefore, there is no component of the electric field in the axial direction of the conductors of the electric field.

The expressions of the scalar and vectorial potentials for the geometry of the case are:

\[
\phi = - \frac{1}{4\pi \epsilon_0} \int_0^\infty \frac{q(z')dz'}{r^2 + (z' - z_0)^2} \quad (6)
\]

\[
A = - \frac{\mu_0}{4\pi} \int_0^\infty \frac{ldz'}{r^2 + (z' - z_0)^2} \quad (7)
\]

where:
- \(hc\) = height of the cloud
- \(q(z')\) = electric charge in the plasmatic channel at altitude \(z'\) at time instant \(t\)
- \(I\) = Magnitude of the return stroke electric current
- \(z_0\) = altitude over the ground plane, where the electric field is being calculated.
- \(r\) = radial distance (xy plane) between the discharge and the point (xo,yo)
- \(Z_0\) = heigth of the point in the plasmatic channel.

The variable \(Z_0\) is denominated retarded heigth of the return stroke over the ground plane, and is given by the relations:

\[
Z_0 = \int_0^t v(t)dt = v.t_0 \quad (8)
\]

\[
t_0 = t - \frac{(Z_0 - Z_0')}{c} \quad (9)
\]

where:
- \(t\) = time after the return stroke start
- \(t_0\) = effective time for the return stroke, with respect to the point where the field is being calculated.

In the calculations, it is assumed that the electric field (vertical) is approximately constant between the ground plane and the conductors height, given the fact that the altitude of the cloud is relatively large in comparison with it. This premise is specially valid for the case of atmospheric discharges that take place near to the electric circuit (< 1 km). In this case, the resultant expressions can be simplified, and the expressions are evaluated at \(z_0=0\) (m):

\[
\left( \frac{\partial \phi}{\partial z} \right)_{z=0} = -\frac{601}{\beta} \left( \frac{1 - \beta^2}{\beta^2 c^2 t^2 + (1 - \beta^2) r^2} \right) \quad (10)
\]

\[
\left( \frac{\partial A}{\partial t} \right)_{z=0} = -\frac{601}{\beta} \left( \frac{\beta^2}{\beta^2 c^2 t^2 + (1 - \beta^2) r^2} \right) \quad (11)
\]
The following premises are the basis for the analysis:

- The electrostatic effect associated to the charge of the cloud, as well as to the building and descent processes of the plasmatic channel of the atmospheric discharge are neglected.
- The discharge channel is assumed to be vertical, with a return stroke wave that starts at the ground plane at $t=0$ sec.
- The conductors of the electric system are considered as ideal conductors. The ground conductivity is assumed with an infinite value.
- The analysis is performed for a general wave shape for the return stroke discharge.
- The formulation cylindrical coordinate system is used, defined as shown in Fig.2.

**Fig 2. Cylindrical Coordinate System**

Master & Uman stated the expressions of the electric and magnetic field at any point of space $(r, \phi, Z)$, produced by a vertical dipole of infinitesimal length $(dZ')$ localized at an altitude $Z'$ in the plasmatic channel of the discharge, by solving the Maxwell equations in terms of the scalar and vectorial retarded potentials:

$$E_r(r, \phi, Z, t) = \frac{dZ'}{4\pi \epsilon_0} \left( \frac{3\pi (Z - Z')}{R^3} \right) \int_0^{\tau'} I(Z', \tau') d\tau' + \frac{3\pi (Z - Z')}{eR^4} - \int_0^{\tau'} \frac{\pi (Z' - Z)}{e^2 R^3} \frac{\partial I(Z', \tau')}{\partial \tau} d\tau'$$

(12)

$$E_z(r, \phi, Z, t) = \frac{dZ'}{4\pi \epsilon_0} \left( \frac{2(Z - Z')^2 - r^2}{R^3} \right) \int_0^{\tau'} I(Z', \tau') d\tau' + \frac{2(Z - Z')^2 - r^2}{eR^4} - \int_0^{\tau'} \frac{\pi (Z' - Z)}{e^2 R^3} \frac{\partial I(Z', \tau')}{\partial \tau} \frac{r^2}{R^3} d\tau'$$

(13)

$$B_\phi(r, \phi, Z, t) = \frac{\mu_0 dZ'}{4\pi} \left( \frac{r}{eR^2} \frac{\partial I(Z', \tau')}{\partial \tau} + \frac{r}{R^3} I(Z', \tau') \right)$$

(14)

where:

- $E_r(r, \phi, Z, t)$ = horizontal component of the electric field
- $E_z(r, \phi, Z, t)$ = vertical component of the electric field
- $B_\phi(r, \phi, Z, t)$ = horizontal component of the magnetic flux density
- $I(Z', \tau')$ = electric current in the plasmatic channel at altitude $Z'$, at time instant $\tau'$
- $\tau' = \text{effective time of the return stroke discharge for given } r, \phi, Z$
- $\tau' = -R/c, \tau' = -R/c, r, Z, R, Z'$ are described in Fig.2.a, 2.b and 2.c.

The ideal ground plane is simulated using the image method, where the field due to the image dipoles is obtained by substituting $R'$ by $R$ and $-Z'$ by $Z$ in equations (12) to (14).

The plasmatic channel is segmented into a number of dipoles by means of an integration in $Z'$. The total field at the point $(r, \phi, Z)$ of the space is obtained by adding the contribution of all the dipoles.

A distribution model for the charge of the plasmatic channel is required, as described in section (a) of the introduction of the article.

C.- *Determination of the induced voltages on the conductors of the electric circuits.*

The evaluation of the induced voltages in the conductors of an electric circuit due to an atmospheric discharge in the vicinity, can be performed in the time domain, using the theory of electric transmission lines, stating and solving the system of wave equations, modified to consider the illumination of the circuit by an external electromagnetic field.
Basically, three arrangements of the system of equations that define the behavior of voltages and currents of a transmission line subject to external electromagnetic fields have been proposed.[2,6,16] They assume ideal conductors, and a ground of finite, constant conductivity.

i.- The formulation proposed by Agrawal [16] introduces the horizontal component axial to the conductors and the vertical component of the electric field as forcing functions exciting the transmission line.

\[
\frac{\partial}{\partial x} V^x(x,t) + [R] \cdot I(x,t) + [L] \cdot \frac{\partial}{\partial t} I(x,t) = E_x(x,z,t)
\]

(15)

\[
\frac{\partial}{\partial x} I(x,t) + [G] \cdot V^y(x,t) + [C] \cdot \frac{\partial}{\partial t} V^y(x,t) = 0
\]

(16)

and, of course:

\[
V(x,t) = V^x(x,t) + V^y(x,t)
\]

(17)

where:

- \( E_x(x,z,t) \): vector of axial components of the electric field for each of the conductors of the circuit. (V/m)
- \( V^x(x,t) \): vector of scaled voltages (V)
- \( I(x,t) \): vector of line currents (A)
- \( V(x,t) \): vector of total conductor voltages (V)
- \( V^y(x,t) \): vector of induced voltages of the conductors of the electric circuit.

\( [ R ], [ L ], [ G ], [ C ] \) = Resistance, inductance, conductance and capacitance matrices of the multi-conductor circuit, respectively, (per units of length).

The induced voltage \( V^y(x,t) \) is calculated for each conductor, integrating the vertical component of the electric field \( E_z(x,z,t) \) between the ground plane and the conductor height. It is a widely used premise, to assume a constant electric field in the integration trajectory, due to the fact that the conductor height is relatively small in comparison to the altitude of the cloud or to the length of the plasmatic channel.

ii.- In the formulation proposed by Taylor et. al., [6] the magnetic field and the vertical electric field are introduced as forcing functions.

\[
\frac{\partial}{\partial x} V^x(x,t) + [R] \cdot I(x,t) + [L] \cdot \frac{\partial}{\partial t} I(x,t) =\]

\[
= \frac{\partial}{\partial t} \int_{y}^{h} B_y(x,y,z) dz + \frac{\partial}{\partial x} \int_{0}^{z} E_y(x,z,t) dz
\]

(18)

\[
\frac{\partial}{\partial x} I(x,t) + [G] \cdot V^y(x,t) + [C] \cdot \frac{\partial}{\partial t} V^y(x,t) = [C] \cdot \frac{\partial}{\partial t} V^z(x,t)
\]

(19)

where:

- \( B_y(x,y,z) \): magnetic field component in quadrature with the electric circuit (y axis).

Both systems of equations are equivalent, (if the resistance and conductance matrices are neglected), given that the magnetic field may be expressed as a function of the horizontal and vertical components of the electric field.[6]

iii.- The formulation used by Chowdhury [2] is similar to (ii), except for the simplification it includes, neglecting the effect of the magnetic field. In theory, this approximation may lead to the underestimation of the overvoltages of the circuit, and to a loss of precision, as reported in [6].

D.- Simulation of the traveling wave phenomena in the electric circuit.

There are two factors to be considered in the calculation of the voltage at determined location of the electric circuit. The first one refers to the voltage directly induced by the electromagnetic fields generated by the atmospheric discharge. This perturbation has a time delay associated to the time required by the electromagnetic fields to travel between the different sources in the plasmatic channel and the specific location in the electric circuit. The second factor refers to the voltage waves that reach the point under study, at the specific location of the electric circuit, coming from other locations of the electric circuit, closer to the atmospheric discharge, and where the induction phenomena has already taken place.

The solution to the wave equations of the electric circuit modified to consider the voltages induced by the electromagnetic fields generated by the atmospheric discharge may be obtained by means of the application of an integration method of differential equations in partial derivatives.


The Finite Element Technique is other, widely used, methodology. [3,6,16]

III. CALCULATION METHODOLOGY

The calculation methodology that was adopted in this work for the simulation of the behaviour of the electric circuit illuminated by an external electromagnetic field, is an adaptation of the technique proposed in [3], denominated Simplified Surge Impedance (SSI), which is based upon the integration by finite differences of the wave equations, derived from the original statement proposed by Taylor, [6] but neglecting the resistance as well as the conductance terms. The structure of the equations for the case of a single conductor circuit is similar to equations (18),(19), with the correspondent simplifications:
\[ \frac{\partial V(x,t)}{\partial x} + L \frac{\partial I(x,t)}{\partial t} = \frac{\partial B_y(x,t)}{\partial t} \ h \]  \hspace{1cm} (20)

\[ \frac{\partial I(x,t)}{\partial x} + C \frac{\partial V(x,t)}{\partial t} = -C \frac{\partial E_v(x,t)}{\partial t} \ h \]  \hspace{1cm} (21)

According to the SSI technique, for each of the segments in which the circuit is divided into, the progressive (forward) and regressive (backwards) travelling waves have two components. The first component refers to the waves coming from the neighboring segments, and the second component is due to the induced currents in the segment due to the atmospheric phenomena.

After expressing the equations in terms of finite differences, and replacing \( V(x,t) \) and \( I(x,t) \) by their equivalents in terms of the forward and backward travelling waves, and considering the approximation \( \Delta t/\Delta x = \sqrt{\frac{1}{LC}} \), the following expressions are obtained, which are the basis of the calculations:

\[ I_f(x,t) = I_f(x-1,t - \Delta t) - \frac{[E_v(x,t) - E_v(x,t - \Delta t)]}{\sqrt{2Z_{\text{onda}}}} \frac{h}{\Delta t} + \frac{[B_y(x,t) - B_y(x,t - \Delta t)]}{\sqrt{2Z_{\text{onda}}}} \frac{h \Delta x}{\Delta t} \]  \hspace{1cm} (22)

\[ I_b(x,t) = I_b(x+1,t - \Delta t) + \frac{[E_v(x,t) - E_v(x,t - \Delta t)]}{\sqrt{2Z_{\text{onda}}}} \frac{h}{\Delta t} + \frac{[B_y(x,t) - B_y(x,t - \Delta t)]}{\sqrt{2Z_{\text{onda}}}} \frac{h \Delta x}{\Delta t} \]  \hspace{1cm} (23)

where:

- \( I_f(x,t) \): forward current wave at location \( x \), time \( t \).
- \( I_b(x,t) \): backward current wave at location \( x \), time \( t \).
- \( Z_{\text{wave}} \): characteristic or wave impedance of the conductor.

The voltage at any time, at a distance \( x \), is defined by:

\[ V(x,t) = Z(\ I_f(x,t), I_b(x,t) ) \]  \hspace{1cm} (24)

The distance \( x \) is expressed in terms of a finite number of circuit segments, \( i = 1, \ldots, n \), so that:

\[ V(i,t) = Z(\ I_f(i,t), I_b(i,t) ) \]  \hspace{1cm} (25)

Where \( i \) is the number of the correspondent circuit segment.

IV. RESULTS

The algorithms used by the authors for the calculations were validated prior to the comparison of the performance of both methods (Master & Uman, Chowdhuri) with results presented in the literature.

In order to validate the results, the Master & Uman calculation procedure, was applied to the test cases presented in [3], where results obtained using the same approach are reported.

The formulation of the Chowdhuri approach was applied and compared to the cases tested in [13].

Results reflected differences between 1% and 4% for the Master & Uman approach and between 1% and 9% for the method of Chowdhuri when the results presented in [3] and [13] were compared with the figures obtained in this work.

In Fig. 3, waveshapes are presented for the voltage developed over the insulation of a single conductor in space, when both simulation techniques are applied, for a 100 kA electric discharge, with a front time of 2 µs, at a distance of 100 m of the circuit.

Similarly, Fig. 4, presents the maximum voltage values detected across the circuit insulation (in per unit of the discharge current), as a function of the transversal distance between the circuit and the point of impact of the atmospheric discharge.
from a difference of 2% (0.14 kV/kA) for discharges at 25 m, to 75% (1.5 kV/kA) for discharges at 200 m and 300% (1.5 kV/kA) for discharges at 500 m.

Also the rate of rise of the front of wave of the voltage obtained by the method of Chowdhury (770 kV/µs) is considerably higher than for the method of Master & Uman (69 kV/µs).

V. CONCLUSIONS

A comparison of the approaches proposed by Chowhuri and Master & Uman was performed reflecting important differences in the results. The first approach showed a tendency to estimate more severe stresses for the circuit insulation, resulting in higher overvoltages, with a higher rate of rise, for the simulated test case. This tendency increased as the distance between the lightning stroke and the electric circuit increases. Therefore significant differences in the behaviour of the insulation of the circuits are expected depending on which of both approaches is applied.

VI. REFERENCES


