Fractional order PI$^{\alpha}$ controller applied to the induction machine current loop

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Abstract—**This paper investigates the use of fractional order PI$^{\alpha}$ controllers, applied to inverter driven induction machines. Fractional order controllers provide additional degrees of freedom for their design, and the non-linearity introduced by the fractional part of the controller provides better tuning for the operation of the controller. In this work a fractional PI controller is applied to the current loop of a vector controlled induction machine and the results are compared to the ones obtained with a conventional PI controller.**

Index Terms—**Induction machine drives, current loop controller, fractional order PI.**

I. Introduction

The current loop control plays an important role in many power electronics applications, specially in the quality and dynamics of PWM based converter operation. Several techniques have been applied over the years such as on-off or bang bang control, deadbeat control, PI control, fuzzy logic, neural networks, variable structure controllers, resonant PI controllers, etc. In general standard PI controllers are often used due to their simplicity and familiarity of the field engineers in their use. In a series of papers around a decade ago [1]–[4], fractional order controllers were put forward to extend the applicability of PI controllers, this last one becoming a particular case of these fractional order PI$^{\alpha}$ controllers.

The induction machine is the most used electric rotating motor, 60% of the electrical energy in industry is consumed by inductor motors [5]. Among its major advantages are the simplicity of its structure which leads to lower manufacturing costs, as well as its ruggedness and low maintenance requirements. As a main drawback, however, it presents a highly nonlinear behavior between stator voltage and developed torque [6].

This paper investigates the use of fractional order PI$^{\alpha}$ controllers for the current loop of a field oriented controlled induction machine under direct current injection.

II. Fractional-order Calculus and controllers

Fractional order calculus was introduced by Leibniz in 1695, when he introduced the order n derivative,

$$\frac{d^n f(x)}{dx^n}, \tag{1}$$

together with L'Hopital they introduced the derivative for $n = 1/2$ [7]. The order of the differential and integral operators can be real, rational or complex.

The fractional operator $D^{\mu}x$ is defined as the order μ derivative, where $\alpha \in \mathbb{R}^+$ [8], Liouville defines an expression for a μ order derivative of an exponential function [7], as follows

$$D^{\mu}(e^{ax}) = a^{\mu}e^{ax} \tag{2}$$

A. Fractional derivative, integral and Laplace transformation

The fractional integral defined by Riemann-Liouville takes the form [9],

$$I^{\alpha}f(t) = \frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha} f(\tau)d\tau, \quad t>0, \alpha \in \mathbb{R}^+ \tag{3}$$

where $\Gamma(\alpha)$ is the Gamma function.

The Laplace transformation of the Riemann-Liouville integral is,

$$\mathcal{L}\{I^{\alpha}f(t)\} = s^{-\alpha}F(s) \tag{4}$$

The Riemann-Liouville fractional derivative is defined as [1],

$$D^{\alpha}f(t) = \frac{1}{m-\alpha}\left(\frac{\partial}{\partial t}\right)^m \int_0^t \frac{f(\tau)}{(t-\tau)^{\alpha-m-+1}}d\tau \tag{5}$$

The Laplace transformation of the Riemann-Liouville derivative is [1],

$$\mathcal{L}\{D^{\alpha}f(t)\} = s^{\alpha}F(s) - \sum_{k=0}^{m-1} s^k [D^{k-\alpha-1}f(t)]_{t=0} \tag{6}$$

where $m-1 < \alpha < m$, $m \in \mathbb{N}$.

B. Classical PID and fractional PI$^{\alpha}$D$^{\mu}$ controllers

The transfer function for a fractional PI$^{\alpha}$D$^{\mu}$ has a proportional gain and fractional integrator and differentiator in the following form [10],

$$G_c(s) = k_p + \frac{k_i}{s^{\alpha}} + k_d s^{\mu}, \quad \alpha, \mu > 0 \tag{7}$$

When the transfer function contains fractional order terms, such as s^{α} and s^{μ}, for α and μ are non integer positive real numbers, the transfer function can be approximated in such a way that the Laplace operator s has integer positive powers. In this way the z-transformation can be used to express in approximate form the differences equation using integer order powers of z.

C. Approximation of fractional order transfer functions in the discrete domain

The discretization is performed using different methods to approximate the s operator, such as the Tustin approximation [11],

$$s \approx \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}} \tag{8}$$

or by using a truncated Maclaurin series,

$$C(z^{-1}) = \left(\frac{2}{T}\right)^{\alpha}\Gamma(\alpha+1)\Gamma(-\alpha+1)\sum_{k=0}^{N} z^{-k}\times \sum_{m=0}^{k}\frac{(-1)^m}{\Gamma(\alpha-m+1)\Gamma(m+1)\Gamma(k-m+1)\Gamma(-\alpha+m-k+1)} \tag{9}$$

III. Use of the fractional PI controller in the current loop of a vector-controlled induction machine with DC injection

In this particular case, a recent application of DC injection for estimating the temperature in an induction machine is used to test the operation of the fractional PI^{α} controller [12]. The machine model, in rotor flux coordinates is as follows,

$$\vec{v}_s^{\Psi_r}(t) = R_s\vec{i}_s^{\Psi_r}(t) + \frac{\vec{\Psi}_s^{\Psi_r}(t)}{dt} + j\frac{d\rho}{dt}\vec{\Psi}_s^{\Psi_r}(t) \tag{10}$$

$$\vec{v}_r^{\Psi_r}(t) = 0 = R_r\vec{i}_r^{\Psi_r}(t) + \frac{d\Psi_r}{dt} + j\left(\frac{d\rho}{dt} - \omega_r(t)\right)\Psi_r(t) \tag{11}$$

where $\rho(t)$ is the rotor flux angle and $\omega_r(t)$ is the mechanical speed. Equation 11 is most commonly written as a function of state variables Ψ_r and $\vec{i}_s^{\Psi_r}$, as follows,

$$\vec{v}_r^{\Psi_r}(t) = 0 = \left[\frac{R_r}{L_r} + j\left(\frac{d\rho}{dt} - \omega_r(t)\right)\right]\Psi_r - \frac{R_r}{L_r}L_m\vec{i}_s^{\Psi_r} + \frac{d\Psi_r}{dt} \tag{12}$$

And, from (12) the direct and quadrature components are,

$$\frac{d\Psi_r}{dt} + \frac{R_r}{L_r}\Psi_r = \frac{R_r}{L_r}L_m i_{sd} \tag{13}$$

$$\frac{d\rho}{dt} - \omega_r(t) = R_r L_m\frac{i_{sq}}{\Psi_r} \tag{14}$$

Equations (13) and (14) provide the current references for the current loops in a rotor flux oriented controller. Figure 1 shows the top level of the Simulink® implementation of the current loop controllers.

IV. Simulated results

For comparison, the system has been simulated using first an standard proportional controller with unit gain, then with a heuristically tuned PI controller, and finally with a hand tuned fractional controller.

A. proportional controller with unit gain

Figure 2 shows the reference and actual currents when the current loop controller is of proportional type with unit gain. The measured current with this controller follows the reference, but there is a noticeable ripple in the measured current around the reference current. It is also noticeable, from Fig. 2, the distortion and DC offset caused by the DC injection used in this application. the error between the references and the real signals is measured using the root mean square error for a set of N points, defined by,

$$\epsilon_{RMS} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\epsilon_i^2} \tag{15}$$

After a 2 seconds simulation, the root mean square error is computed for the last 500 ms of the error in each a, b, and c phases. Table I shows the root mean squared error for each phase.

B. Standard PI controller

An standard PI controller heuristically tuned, with the following parameters is employed in the current loop of the induction machine. The parameters for the PI controller are: K_p=0.1 and K_i=210. The results of the simulations in this case are shown in Fig. 3, and the root mean square for the current loop with the classical PI controller is shown in Table II.

C. Fractional order PI^{α} controller

A hand tuned fractional order PI^{α} controller is designed using the "*nipid*" function, from the **NINTEGER** toolbox reported in [13]. This function provides both, continuous and discrete approximations of a transfer function in the following form,

$$C(s) = k_P + k_D s^{\mu} + \frac{k_I}{s^{\alpha}}, \quad \mu, \alpha \in \mathbb{R}^+ \tag{16}$$

The controller employed in this work was designed using the *nipid* function with the following parameters:

TABLE I: Root mean squared error for the unit gain proportional controller in current loop.

Phase	Root Mean Square Error
a	4.8081 A
b	4.8220 A
c	4.8126 A
average	4.8142 A

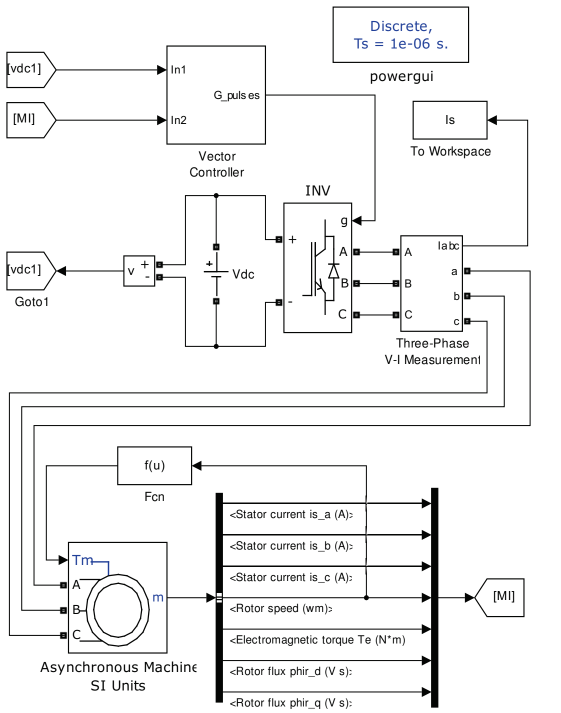

Fig. 1: Top level for the Simulink implementation of the rotor flux oriented vector controller.

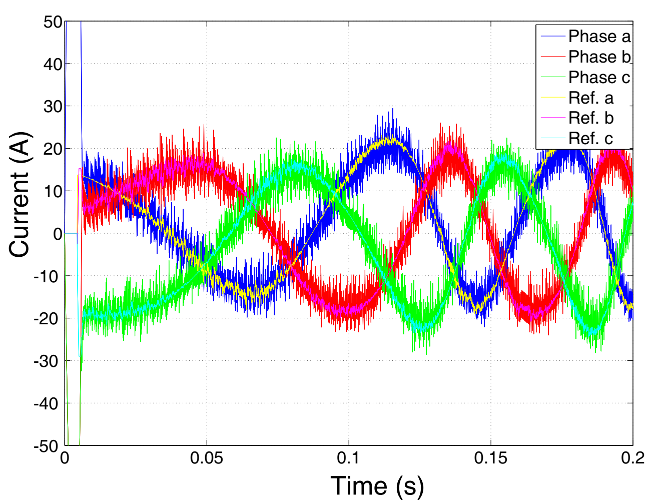

Fig. 2: Phase currents and references for the current loop with a proportional controller at unit gain.

```
Kp=0.075; Ki=190; alpha=0.81;
Kd=0; Mu=0;
T = 100e-6; % Sampling time

Gc = nipid(Kp, Kd, Mu, Ki,...
alpha,[T; 1000],4,...
'tustin','mcltime', 'frac')
```

This results in the following fifth order approximated controller,

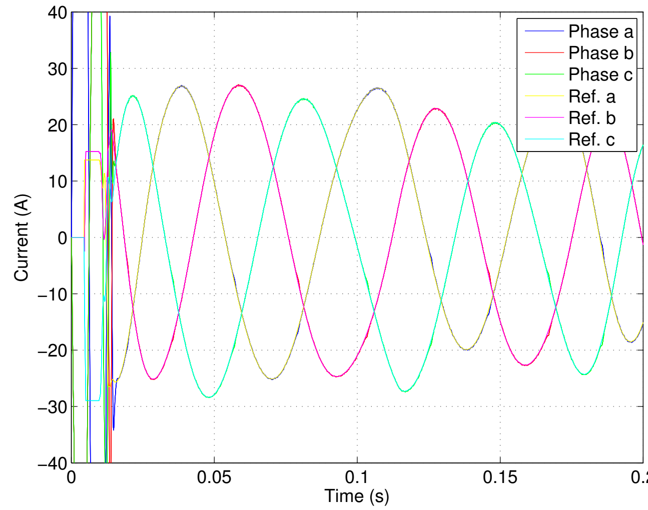

Fig. 3: Phase currents and references for the current loop with a standard PI controller.

$$G_c = \frac{0.2747 - 0.0727z^{-1} - 0.0384z^{-2}}{2(1-z^{-1})} - \frac{0.0079z^{-3} + 0.0108z^{-4} - 0.0061z^{-5}}{2(1-z^{-1})} \quad (17)$$

Equation 17 written as difference equations becomes,

$$y_k = \frac{0.2747}{2}x_k - \frac{0.0727}{2}x_{k-1} - \frac{0.0384}{2}x_{k-2} - \frac{0.0079}{2}x_{k-3} - \frac{0.0108}{2}x_{k-4} + \frac{0.0061}{2}x_{k-5} - y_{k-1} \quad (18)$$

Table III shows the roor mean squared error obtained for this controller in the current loop.

D. *Fractional order PI$^{\alpha}$ controller for α={0.72, 0.50 and 0.30}*

Using a procedure similar to the one in the previous subsection, the value for α is modified, as well as the proportional and integral constants. Table IV shows the root

TABLE II: Root mean squared error for the classical PI controller in the current loop.

Phase	Root Mean Square Error
a	0.6153 A
b	0.5779 A
c	0.6020 A
average	0.5984 A

TABLE III: Root mean squared error for the fractional order PI$^{\alpha}$ controller, α=0.81, in the current loop.

Phase	Root Mean Square Error
a	0.2484 A
b	0.2394 A
c	0.2416 A
average	0.2431 A

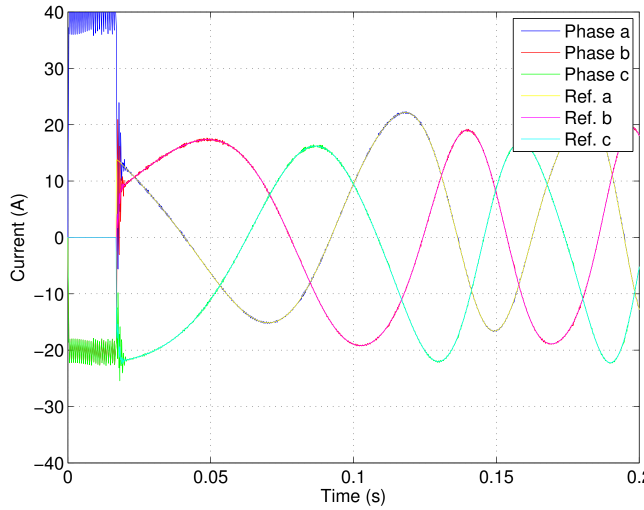

Fig. 4: Phase currents and references for the current loop with the fractional order PI$^\alpha$ controller.

mean squared error for the fractional order PI$^\alpha$ controller with α=0.72, k_P=0.085 and k_I=70. Table V shows the root mean squared error for the fractional order PI$^\alpha$ controller with α=0.50, k_P=0.095 and k_I=5. Table VI shows the root mean squared error for the fractional order PI$^\alpha$ controller with α=0.30, k_P=0.12 and k_I=0.055.

Table VII summarizes the average over all phases, of the root mean squared error for each controller in the current loops.

E. Effect of stator and rotor resistance change on the current loop controllers

The current loop controller studied in this paper is intended to be used for DC current injection in induction machines, for temperature estimation. The main effect of temperature variation is on the stator and rotor resistances, so the current loop controllers are analyzed to verify tuning during changes in the stator and rotor resistances.

The design parameters in all the controllers are kept constant during the change in the stator and rotor resistances. The system is simulated during two seconds and the root mean squared error is computed during the last 500 ms. The values of the three phases are averaged. Table VIII shows the root mean squared error for a 50%, 100% and 200% change in stator resistance, Table IX shows the root mean squared error for a 50%, 100% and 200% change in rotor resistance, and Table X shows the root mean squared error for a 50%, 100% and 200% simultaneous change in stator and rotor resistances. It is clear from these tests that the current loop controller is not very sensitive to large changes in stator and rotor resistances, hence the fractional order PI$^\alpha$ controller provides a good solution for the tracking of current references.

TABLE IV: Root mean squared error for the fractional order PI$^\alpha$ controller, $\alpha = 0.72$, in the current loop.

Phase	Root Mean Square Error
a	0.2794 A
b	0.2642 A
c	0.2739 A
average	0.2725 A

TABLE V: Root mean squared error for the fractional order PI$^\alpha$ controller, $\alpha = 0.50$, in the current loop.

Phase	Root Mean Square Error
a	0.5024 A
b	0.4722 A
c	0.4925 A
average	0.4890 A

TABLE VI: Root mean squared error for the fractional order PI$^\alpha$ controller, $\alpha = 0.30$, in the current loop.

Phase	Root Mean Square Error
a	2.9514 A
b	2.9122 A
c	2.9753 A
average	2.9463 A

TABLE VII: Summary of the root mean squared error in the controllers.

Controller	Root Mean Square Error phase average
Proportional, k_P=1	4.8142 A
Classical PI	0.5984 A
Fractional PI$^\alpha$, α=0.81	0.2431 A
Fractional PI$^\alpha$, α=0.72	0.2725 A
Fractional PI$^\alpha$, α=0.50	0.4890 A
Fractional PI$^\alpha$, α=0.30	2.9463 A

TABLE VIII: Root mean squared error in the current loop, for different controllers with changes in stator resistance.

	root mean square error		
Controller	SRI 50%	SRI 100%	SRI 200%
proportional k_P=1	24.08%	24.26%	24.34%
classical PI	3.07%	3.16%	3.31%
Fractional PI$^\alpha$, α=0.81	1.24%	1.26%	1.30%
Fractional PI$^\alpha$, α=0.72	1.4%	1.42%	1.49%
Fractional PI$^\alpha$, α=0.50	2.51%	2.58%	2.71%
Fractional PI$^\alpha$, α=0.30	15.01%	15.29%	15.84%

V. Conclusion

In this paper several known controllers have been tested for the current loop control in an induction ma-

TABLE IX: Root mean squared error in the current loop, for different controllers with changes in rotor resistance.

	root mean square error		
Controller	RRI 50%	RRI 100%	RRI 200%
proportional k_P=1	24.28%	24.42%	25.00%
classical PI	3.30%	3.61%	4.27%
Fractional PI$^\alpha$, α=0.81	1.30%	1.41%	1.63%
Fractional PI$^\alpha$, α=0.72	1.5%	1.63%	1.91%
Fractional PI$^\alpha$, α=0.50	2.7%	2.95%	3.5%
Fractional PI$^\alpha$, α=0.30	15.56%	16.44%	18.05%

TABLE X: Root mean squared error in the current loop, for different controllers with changes in stator & rotor resistances.

	root mean square error		
Controller	SR_RI 50%	SR_RI 100%	SR_RI 200%
proportional k_P=1	24.32%	24.64%	24.75%
classical PI	3.38%	3.78%	4.62%
Fractional PI$^\alpha$, α=0.81	1.32%	1.46%	1.75%
Fractional PI$^\alpha$, α=0.72	1.53%	1.70%	2.07%
Fractional PI$^\alpha$, α=0.50	2.76%	3.09%	3.78%
Fractional PI$^\alpha$, α=0.30	15.83%	17.00%	19.15%

chine subjected to DC injection. The injection of DC current imposes an additional nonlinearity to the highly nonlinear equations of the induction machine. By using fractional order PI$^\alpha$ controllers a more stable tracking of current references can be obtained if compared with classic proportional and proportional-integral controllers. The effect of stator and rotor resistance change on the current loop controllers was also tested resulting in a better performance for the PI$^\alpha$ controller. The additional variable α in the fractional order PI$^\alpha$ controller provides also and additional degree of freedom in tuning the current loop controller.

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