A Novel Analytic Wavelet Ridge Detector for Dynamic Eccentricity Detection in BLDC Motors under Dynamic Operating Conditions

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Abstract – **A new method using the analytic wavelet transform of the stator current signal is proposed for detecting dynamic eccentricity in Brushless Direct Current (BLDC) motors operating under rapidly varying speed conditions. As wavelets are inherently suited for non-stationary signal analysis, this method does not require the use of any windows nor is it dependent on any assumption of local stationarity. The time-frequency resolution obtained is therefore better than other existing techniques such as the short time Fourier transform (STFT). Experimental results are provided to show that the proposed method works over a wide speed range of motor operation and provides an effective and robust way of detecting rotor faults such as dynamic eccentricity in BLDC motors.**

I. INTRODUCTION

Brushless Direct Current (BLDC) motors are rapidly gaining popularity. BLDC motors are now widely used in industries such as appliances, automotive, aerospace, consumer, medical, industrial automation equipment and instrumentation. As the name implies, BLDC motors do not use brushes for commutation; instead, they are electronically commutated. BLDC motors have many advantages over brushed DC motors and induction motors. A few of these are: better speed versus torque characteristics, higher dynamic response, higher efficiency, and noiseless operation. In addition, the ratio of torque delivered to the size of the motor is higher, making it useful in applications where space and weight are critical factors [1]. More BLDC machines are therefore being used, often in critical high performance applications. Fault diagnosis and condition monitoring of BLDC machines is therefore assuming a new importance. Early detection of faults and asymmetries would allow preventive maintenance to be performed and provide sufficient time for controlled shutdown of the process, thereby reducing the costs of outage time and repairs. The range of faults on BLDC machines includes stator faults, rotor faults, and inverter faults.

Diagnostics of electrical motors operating under constant speed conditions has been well investigated in literature [2-4]. The fundamental assumption of stationarity in constant speed operation allows the use of the well known method of Fourier Transformation in the frequency analysis of currents, voltages, and vibration signals to detect various rotor and mechanical faults in an electrical machine. However, there are several applications where the motor is never operating at a constant speed or load. The motor operating in such a non-stationary environment has a non-stationary voltage, current and vibration signal. Some attempts have been made to detect machine faults in motors operating under different cases of slowly varying load and speed conditions. In [5] the authors use Short Time Fourier Transforms (STFT) and pattern recognition techniques to detect faults in induction motors operating in varying operating conditions. Their application is however based on the assumption that the change in speed and load occur very slowly and there are sufficiently long intervals of time where the motor can be assumed to operate in a stationary condition. This is common in applications such as rolling steel mills and the cement industry. Another approach to a similar application, through the use of wavelets, is presented in [6]. Another area where non-stationary signal analysis has been attempted is in the diagnosis of machine faults from the starting current transients in induction motors. This enables the detection of faults under no load condition where the only measurable and useful current exists during the starting of the motor and is in a transient state [7,8]. However, none of these methods attempt to diagnose faults in applications where the motor is continuously in a non-stationary stage or where assumptions of local or slow stationarity may be questionable and unrealistic.

A novel Analytic Wavelet Transform (AWT) based detector for tracking rotor faults such as dynamic eccentricity in BLDC motors is proposed in this paper. The proposed method extracts the rotor fault frequencies from the stator current of the BLDC motor using the AWT. A ridge detector is then applied to track the maxima of the wavelet-extracted fault frequencies over time, thus providing a method to detect rotor faults in motors operating under non-stationary conditions.

II. CHOOSING THE ANALYTIC WAVELET

The analysis of rapidly time varying signals requires detection of time varying events as well as identifying the behaviour of the signal over a length of time. One method commonly used to achieve this is the STFT where a short length of the signal is analyzed at a time. The use of a single window is thus a compromise between time and frequency resolution. However, in wavelet analysis, a signal is analyzed at different scales or resolutions: a large window is used to look at the approximate stationarity of the signal and a small window is simultaneously used to look for transients. This multi-resolution or multi-scale view of the signal is the essence of wavelet analysis [9]. The wavelet analysis is performed using a single prototype function called a wavelet. This function is analogous to the sine function used in Fourier transforms but is structured to suit transient applications. Fine temporal analysis is performed using the

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compressed version (high-frequency) of the wavelet while fine frequency analysis uses dilated versions (low-frequency) of the wavelet [9]. The wavelet transform decomposes signals over dilated and translated wavelets. A wavelet is a function ψ with a zero average [10].

$$\int_{-\infty}^{+\infty} \psi(t)\,dt = 0\,, \tag{1}$$

It is normalized $\|\psi\| = 1$, and centred in the neighbourhood of t = 0. A family of time-frequency atoms is obtained by scaling ψ by s and translating it by u:

$$\psi_{u,s}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-u}{s}\right) \tag{2}$$

The wavelet transform of a signal $f(t)$ at time u and scale s is given by

$$Wf(u,s) = \langle f, \psi_{u,s} \rangle = \int_{-\infty}^{+\infty} f(t)\frac{1}{\sqrt{s}}\psi^{*}\left(\frac{t-u}{s}\right)dt\,. \tag{3}$$

The energy density of the signal is represented by the scalogram $P_W f$ given by [10]

$$P_W f(u,\xi) = \left|Wf(u,s)\right|^2, \tag{4}$$

where $\xi = \eta/s$ is the centre frequency of the scaled wavelet $\psi_{u,s}$ and η is the centre frequency of the original mother wavelet ψ. A real wavelet does not possess separate phase information and hence is suitable only for detecting sharp transients [10]. However, a complex analytic wavelet can separate amplitude and phase components from a signal and is commonly used to measure instantaneous frequencies [11]. The analytic wavelet is therefore chosen for this application.

III. ANALYTIC WAVELET RIDGES

An AWT is calculated by using an analytic wavelet ψ in (3). Its time-frequency resolution depends on the time-frequency spread of the wavelet atoms $\psi_{u,s}$. An analytic wavelet $\psi(t)$ can be constructed by modulating the frequency ($exp(i\eta t)$) with a real and symmetric window $g(t)$.

$$\psi(t) = g(t)e^{i\eta t}\,, \tag{5}$$

A common analytic wavelet is the Gabor wavelet. It is obtained by frequency modulating a Gaussian window $g(t)$,

$$g(t) = \frac{1}{\left(\sigma^2\pi\right)^{1/4}} e^{-\frac{t^2}{2\sigma^2}}\,, \tag{6}$$

where σ is the variance. The Fourier transform of this window is

$$\hat{g}(\omega) = \left(4\pi\sigma^2\right)^{1/4} e^{-\frac{\sigma^2\omega^2}{2}}\,. \tag{7}$$

where ω is the instantaneous frequency. If $\sigma^2\eta^2 \gg 1$ then $\hat{g}(\omega) \sim 0$ for $|\omega| > \eta$. Such Gabor wavelets are thus considered to be approximately analytic [10].

The instantaneous frequencies in the signal f are measured from the ridges of the normalized form of the scalogram $P_W f$ computed using the analytic wavelet. The instantaneous frequency is defined as a positive derivative $\varphi'(t)$ of the phase $\varphi(t)$ of the respective spectral component. The normalized scalogram is defined by

$$\frac{\xi}{\eta}P_W f(u,\xi) = \frac{\left|Wf(u,s)\right|^2}{s}\,, \tag{8}$$

for $\xi = \eta/s$. It can be shown that the scalogram is maximum at [10]

$$\frac{\eta}{s(u)} = \xi(u) = \varphi'(u)\,, \tag{9}$$

The corresponding points $(u,\xi(u))$ are called wavelet ridges. Thus the ridges are the local maxima of the scalogram. In multi-component signals that have more than one frequency, the instantaneous frequencies can be discriminated as long as the bandwidth relations (10) and (11) are satisfied [10] by

$$\frac{\left|\varphi_i'(u) - \varphi_j'(u)\right|}{\varphi_i'(u)} \geq \frac{\Delta\omega}{\eta} \tag{10}$$

and

$$\frac{\left|\varphi_i'(u) - \varphi_j'(u)\right|}{\varphi_j'(u)} \geq \frac{\Delta\omega}{\eta} \tag{11}$$

where $\Delta\omega$ is the bandwidth of $\hat{g}(\omega)$. φ_i and φ_j are any two frequency components in the given signal. Equations (10) and (11) are used to select the bandwidth and scales for the wavelet. The AWT can be practically implemented in real-time and integrated into a motor drive system using a simple and fast method based on the well known Fast Fourier Transformation (FFT) [11].

IV. DYNAMIC ECCENTRICITY IN BLDC MOTORS

Rotor faults in BLDC machines are usually comprised of eccentricities (dynamic and static) and chipped rotor magnets [12]. Machine eccentricity is the condition of unequal air-gap that exists between the stator and the rotor. Air-gap eccentricity can occur in the form of static or dynamic eccentricity. In the case of static eccentricity, the position of minimum radial air-gap length is fixed in space. Typical causes of static eccentricity include stator core ovality or incorrect positioning of the rotor or the stator at assembly stage. Dynamic eccentricity occurs when the centre of the rotor is not at the centre of rotation and the minimum air-gap revolves with the rotor. This means that dynamic eccentricity is a function of space and time. Typical causes of dynamic eccentricity include bent shaft, mechanical resonances at critical speeds, or bearing wear.

When eccentricity becomes large, the resulting unbalanced radial forces (also known as unbalanced magnetic pull or UMP) can cause stator-to-rotor rub, and this can result in damage to the stator and the rotor [3]. In the case of static eccentricity this is a steady pull in one direction. This makes the UMP difficult to detect unless specialized equipment is used, which is not possible for motors in service [4]. Dynamic eccentricity on the other hand produces a UMP which rotates at the rotational speed of the motor and acts directly on the rotor. This makes the UMP easier to detect by vibration or current monitoring. Due to this reason, dynamic eccentricity is the main focus of this paper.

Researchers in [12] have shown that dynamic eccentricity in BLDC motors operating at a constant speed affect certain characteristic frequency components in the machine stator current and this rotor defect can be identified by monitoring the amplitude of these harmonic components [12]. The frequency spectrum of the stator current is computed using a fast Fourier transform (FFT) [13]. The rotor fault frequencies are then identified from the current spectrum and monitored to detect the severity of the fault. A dynamic eccentricity in BLDC motors causes current components at frequencies given by [12],

$$f_{rf} = f_e \pm k \frac{f_e}{P/2}, \quad (12)$$

where f_{rf} is the rotor fault frequency, f_e is the fundamental frequency, P is the number of poles in the BLDC machine, and k is any integer. These fault frequencies are also present when the BLDC stator current is in a non-stationary state. The health of the BLDC motor can be diagnosed by monitoring these fault frequencies.

V. AWT RIDGE BASED BLDC ROTOR FAULT DETECTION ALGORITHM

The AWT ridge based BLDC fault detection strategy for detecting dynamic eccentricities in a BLDC motor is shown in Figure 1. The sampled stator current is adaptively filtered to remove the fundamental and all harmonics above two. The AWT with Gabor wavelet is then used to compute the scalogram of the current signal. The instantaneous fault frequencies are then extracted from the scalogram by using the wavelet ridge algorithm. The wavelet ridges are the instantaneous local maxima of the scalogram that also possess stationary phase characteristics as explained previously. The AWT ridge extraction is shown in more detail in Figure 2. The amplitudes of the extracted ridges can be monitored to estimate the health of the BLDC motor. A simple fault metric proposed in this paper is the root mean square (RMS) of the instantaneous ridge amplitudes. A heuristically set threshold classifies the motor as good or bad. More sophisticated fault classifiers that set a fault threshold based on the load and speed of the motor can be developed using Artificial Neural Networks.

VI. EXPERIMENTAL ARRANGEMENT

A 6-pole, 12 V, 1 kW BLDC motor is used to implement dynamic eccentricity. The BLDC motor has a current control loop and a speed control loop. The dynamic eccentricity is implemented by first removing the bearing from the shaft. The shaft is then machined off at one end of the motor and shims are now inserted between the bearing and the shaft as shown in Figure 3. The length of the air-gap being 0.75 mm, the rotor is shifted by approximately 0.25 mm (32% of normal air-gap length) from its center by inserting a shim of thickness 0.001". The defect induced in this manner realistically represents a practically occurring fault.

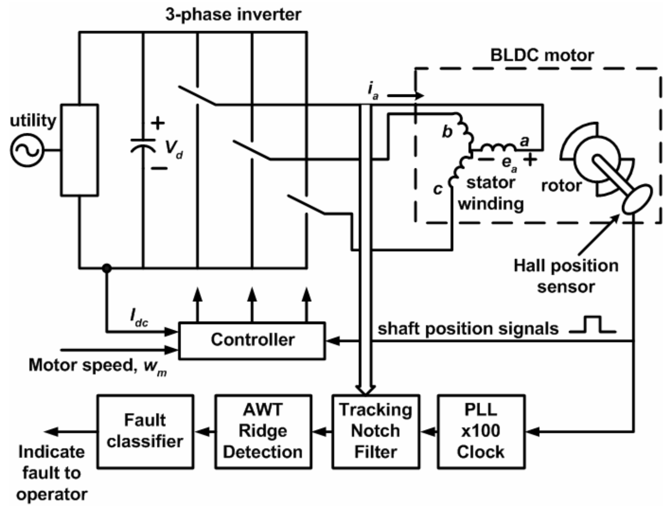

Fig. 1. AWT ridge based BLDC rotor fault detector.

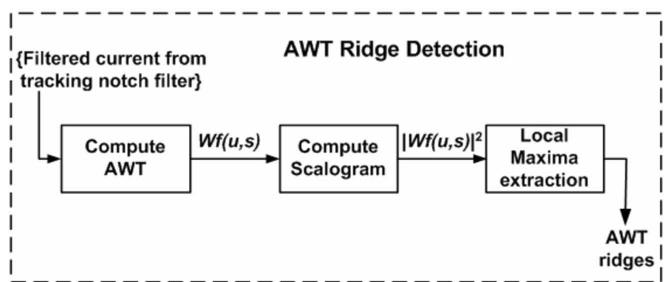

Fig. 2. AWT ridge detection algorithm.

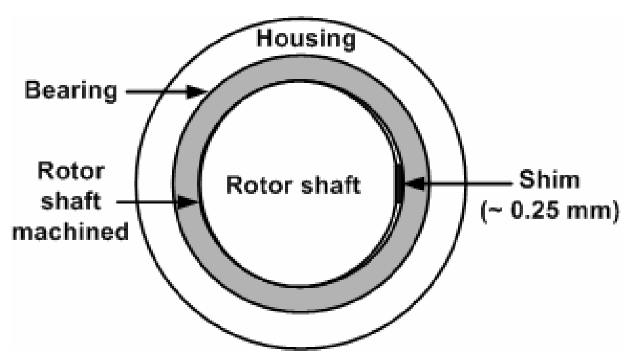

Fig. 3. Implementation of dynamic eccentricity.

The experimental arrangement is shown in Figure 4. The current in one phase of the BLDC motor is acquired using Hall Effect sensors. The fundamental and all harmonics above the third are adaptively filtered using a switched capacitor filter developed in the laboratory. The notch filter used to filter the fundamental is a 6th order Elliptic switch capacitor filter designed using the software FilterCAD from Linear Technologies. Such a high order filter was designed to obtain a sharp notch that would filter only the fundamental and leave the fault frequencies near the fundamental unaffected. The 6^{th} order filter is implemented using the LTC1061 high performance triple universal integrated circuit and provides a notch attenuation of 56 dB. A phase locked loop (PLL) is used to track the fundamental frequency from the BLDC Hall position sensor signal and provide the desired clock to the switch capacitor filter. The filtered and unfiltered currents are sampled at a rate of 2 kHz and acquired using a 16-bit data acquisition system.

Rapid time-varying motor operation is obtained by varying the speed reference. Experiments are conducted with sinusoidal, triangular, and randomly changing speed references. The sinusoidal and triangular references vary at a rate of 0 to 10 Hz representing most practically occurring applications.

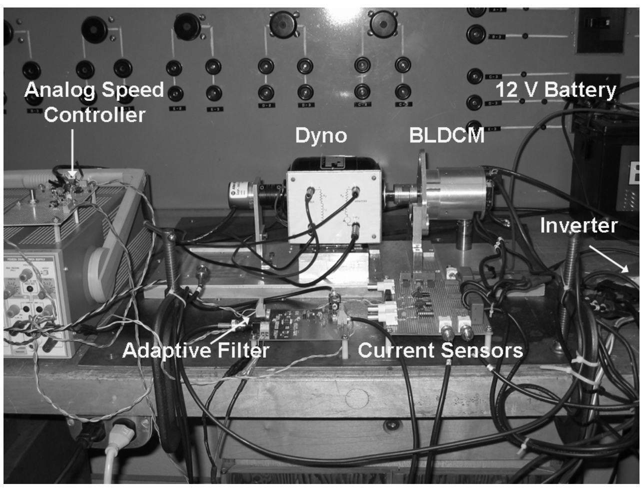

Fig. 4. Experimental arrangement for testing dynamic eccentricity.

VII. EXPERIMENTAL RESULTS

The typical stator current of a BLDC motor operating under non-stationary conditions is shown in Figure 5. For a six-pole BLDC motor, the rotor fault frequencies occur at $1/3^{rd}$, $2/3^{rd}$, $4/3^{rd}$, and $5/3^{rd}$ times the fundamental frequency (3). The AWT ridge algorithm is implemented in MATLAB using the Wavelab802 toolbox from Stanford University [14]. The discrete wavelet version of the AWT is computed at eight scales selected as powers of two, $s^{-1} = \{2,4,8,16,32,64,128,256\}$. Twelve intermediate scales are also computed between each scale. The variance σ of the analytic Gabor wavelet is chosen as one. The frequency parameter η is varied arbitrarily to obtain a good frequency resolution with as little interference terms as possible. A η of 14 is found to offer a good frequency resolution while also maintaining good time resolution.

The stator current of the dynamically eccentric BLDC motor operating at a 5 Hz triangular speed reference and the scalogram ($|Wf(u,s)|^2$ in Figure 2) of the filtered stator current are shown in Figure 6. Darker zones in the scalogram represent higher amplitudes. The local maximas (also called as the AWT ridges) are computed from the scalogram as depicted in Figure 2. These AWT ridges are the instantaneous frequencies of the signal. The instantaneous fault frequencies (AWT ridges) in the filtered stator current of Figure 6 are shown in Figure 7. The two dynamic eccentricity frequencies at $2/3^{rd}$ and $4/3^{rd}$ the fundamental frequencies are distinctly seen to vary over time. The ridge algorithm efficiently extracts only the fault frequencies while suppressing noise and other artifacts. The amplitude of these AWT ridges can be used to measure the health of the motor.

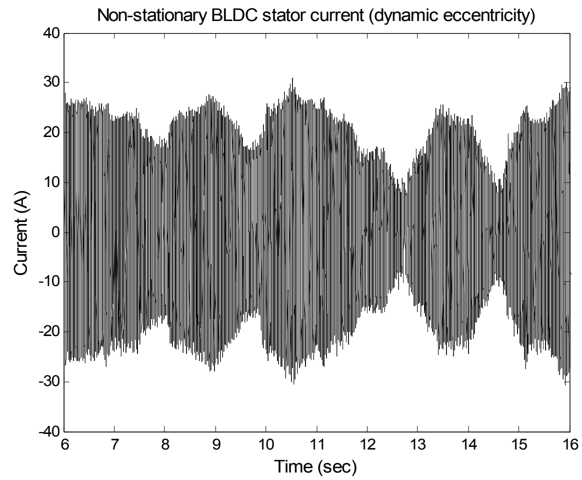

Fig. 5. Non-stationary stator current of a BLDC motor (with dynamic eccentricity).

The AWT ridges of the filtered stator current of a dynamically eccentric BLDC motor now operating with an 8 Hz triangular speed reference are shown in Figure 8. Here again, the AWT ridges are extracted from the scalogram (not shown here for this case) of the filtered stator current as explained in Figure 2. Similarly, the AWT fault ridges of the filtered stator current of a dynamically eccentric BLDC motor operating with a random speed reference (Figure 5) are shown in Figure 9. Some interference terms are seen to be present in Figure 9. These interference terms arise as frequency resolution conditions (10) and (11) are not satisfied for the value of η, chosen here to be 14. These components are however small and do not affect the rotor fault classification. The interference terms can be removed by carefully fine tuning the value of η. The fault ridges are seen to be tracked distinctly over time, thus confirming that AWT is a suitable tool for detecting faults in motors operating under rapidly varying speed conditions.

Once the AWT ridges have been extracted, the last step in a motor dynamic eccentricity detection algorithm is fault classification (Figure 1). A simple fault metric is computed from the amplitudes of the instantaneous fault frequencies (AWT ridges) extracted from the scalogram of the filtered stator current. The fault metric is calculated by obtaining the RMS value of the AWT ridge amplitudes for every instant of time. Only the AWT ridges in a selected frequency band are used to calculate the RMS value. In the present fault scenario, the RMS is calculated for all ridges in a frequency band of $f_e \pm f_e/2$, where f_e is the fundamental frequency of the BLDC stator current. This fault metric is used in a fault classifier that will indicate a motor fault if the metric is above a preset threshold. Figure 10 shows the RMS fault metrics

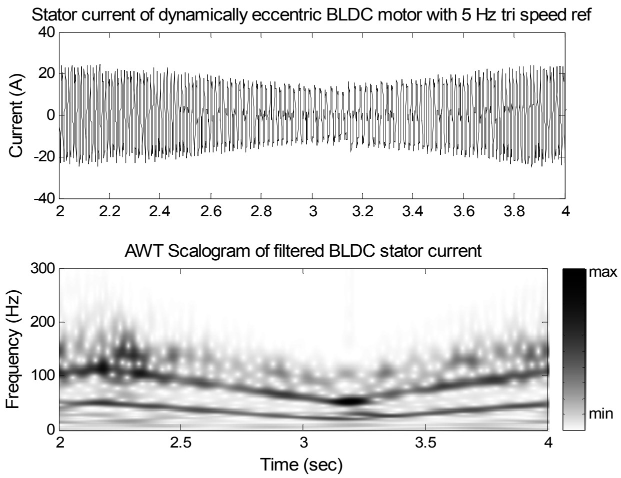

Fig. 6. AWT scalogram of filtered BLDC motor (with dynamic eccentricity) stator current with 5 Hz triangular speed reference.

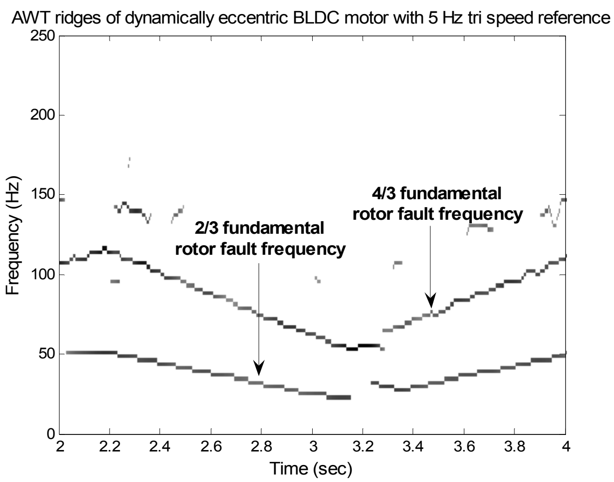

Fig. 7. AWT ridge extraction from filtered stator current scalogram in BLDC motor operating with 5 Hz triangular speed reference.

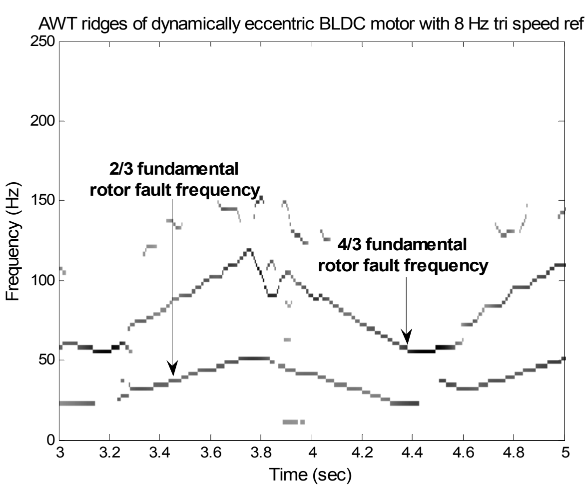

Fig.8. AWT ridges of filtered stator current in BLDC motor operating with 8 Hz triangular speed reference.

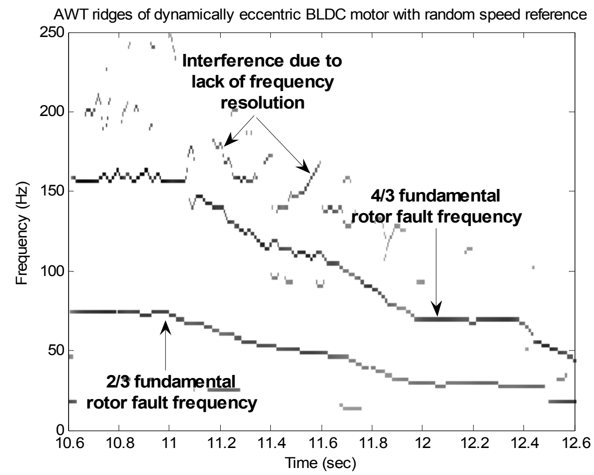

Fig. 9. AWT ridges of filtered stator current in BLDC motor operating with continuous random speed reference.

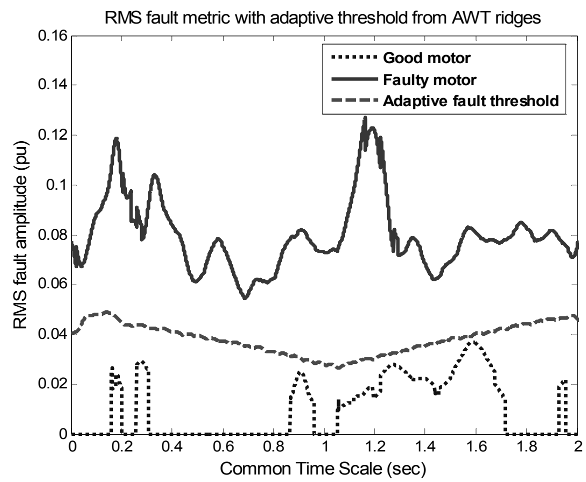

Fig. 10. AWT ridge based BLDC rotor fault detector using RMS fault metric.

computed from the AWT ridge amplitudes of the filtered stator currents of a good and a faulty motor operating with the a 5 Hz triangular speed reference. A clear difference in the fault amplitudes between a good and a faulty motor are seen. A preset adaptive threshold that varies as 2.5% of the fundamental amplitude is set heuristically. It can be seen from Figure 10 that the adaptive thresholding scheme effectively distinguishes a faulty motor from a good one. A common time scale is used as the stator current from the good and the faulty motor were recorded at different times.

VIII. CONCLUSIONS

A novel analytic wavelet transform based detector has been proposed for detection of dynamic eccentricity in BLDC motors operating under never steady state conditions. The method does not need assumption of local stationarity in the signal as wavelets are multi-resolution tools developed for the analysis of non-stationary signals. Experimental results have shown that the AWT ridge method can detect dynamic eccentricity under various types of non-stationary motor operation. A fast wavelet algorithm can be used to implement the method in real-time. The proposed method is not limited to the detection of only dynamic eccentricity, but can also be used for the detection of other faults such as bearing faults. The method can also be used for rotor fault detection in other types of motors such as induction motors besides BLDC motors.

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