Abstract—In this work a parameter estimation method of the induction machine model based on the instantaneous impedance is presented. The proposed method is based on instantaneous voltage and current measurements during start-up of the induction motor at rated voltage. The proposed parameter estimation method uses an space vector description of the transient machine equations, referred to the stator frame of reference, during the dynamic conditions presents at start-up. The method adjusts the dynamic model parameters using an error function obtained by comparing the instantaneous impedance obtained from the space vector representation of the machine and the corresponding obtained from the measured variables at start-up. The method gives a very precise off-line parameter estimation. The solution is obtained using a non-linear regression method that can be used to improve the control behavior in several predictive DTC strategies.

Index Terms—induction machine; parameter estimation; instantaneous impedance

I. INTRODUCTION

The development of vector control theory and more specifically the field oriented method proposed by Blaschke [1], Gabriel et al [2] and others, had made possible many speed control applications of the induction machine, limited in the past only for DC motors. Vector controllers improve the dynamic response performance in comparison with traditional scalar methods. During decades research activity in vector control applications of the induction machine were directed to the accurate estimation of the non-measurable variables in real time. Some authors such as Bellini et al [3], Garcés [4], and Nordin et al [5] have proposed several estimation methods to solve this problem. In general, these methods are dependent on the model parameters, and in general these parameters are varying during normal operation of the machine.

The simplest method for parameter estimation is the classical no load and full-load rotor tests of the induction motor. Using the equivalent circuit of the induction machine in balanced, sinusoidal and steady state operation, an impedance function can be obtained. This input impedance is function of the equivalent circuit parameters, the electric frequency and the rotor slip. With these simplifying assumptions, applied in the transformer circuit parameter estimation, an analogous method can be used for induction machines. Some of these assumptions are not completely applicable in practical applications. Nevertheless, the transformer based parameter estimation method gives an initial and feasible solution.

A second approach can be obtained by proposing an error function obtained by adding the square impedance mismatch between the measurements and the mathematical model of the induction machine, using at least three independent operational points (i.e., no load, blocked rotor, and nominal slip). The absolute minimization of this error function produces the best parameter estimation for this set of operation points, if the model is physically feasible. The non-linear regression involved in the minimization of quadratic functions presents a considerable computational load that precludes its use in real time control applications.

There is a non linear relation between the machine parameters and the electrical frequency, described by the input impedance. The instantaneous time response has a linear relation between parameters and state variables when a constant or quasi-constant rotor speed is considered. Moons and Moor [6], [7], and Stephan et al [8] have proposed a real time estimation of the parameters using linear regression based on the second and third time derivatives of the stator currents and second time derivatives of the stator voltage. This identification method uses the quasi-stationary induction machine equations [9]. The principal problem in this estimation method is the noise introduced by the large order derivative of the physical measurements [10], [11]. The constant or quasi-constant rotor speed constraint does not apply during the start-up of the induction machine [12], [13], and the method does not provide reliable parameters in that case.

The linear regression can be applied for constant or quasi-constant rotor speed operation. Since a non linear regression is proposed in this paper, together with the use of the instantaneous impedance \( z(t) = \frac{\tilde{e}_s(t)}{\tilde{i}_s(t)} \) during the start-up of the induction machine, the constant or quasi-constant rotor speed restriction is relaxed. In this case, the instantaneous impedance is adjusted in a least squares sense, to the instantaneous values of the impedance calculated with the measured voltage and currents. The algorithm includes the speed estimation based on the electric torque computation during the no load machine start-up. The instantaneous electric torque is computed using the corresponding measured voltage and currents during the test.

Using the induction machine model in the stator coordinates frame, the instantaneous impedance equation is derived. Each impedance value obtained with measured variables is com-
pared with the same one obtained from the dynamic model and used to compute the least square error function. The results obtained during the experimental tests indicate that the proposed method has several advantages over the quasi static approach, especially when a non on-line estimation is required. In this work only simulation results are shown and in a future paper the experimental validation will be presented.

II. INDUCTION MACHINE MODEL

The space vector model of the induction machine in the stator frame of reference $\alpha\beta$, could be represented as [14],

$$\vec{v}_s = R_s\vec{i}_s + L_s p\vec{i}_s + L_{sr} p\vec{i}_r$$  \hspace{1cm} (1)

$$0 = R_s \vec{i}_r + L_{sr} p\vec{i}_r + L_r p\vec{i}_r - jn_p \omega_m \left(L_{sr}\vec{i}_s + L_s \vec{i}_r\right)$$  \hspace{1cm} (2)

$$J_p \omega_m = T_c - T_m = n_p L_{sr} \Re\left(\vec{v}_s^* \vec{i}_r\right) - T_m$$  \hspace{1cm} (3)

where, $p = \frac{d}{dt}$, $j = \sqrt{-1}$, $\vec{i}_s$ e $\vec{i}_r$ are the stator and rotor space vectors, $\vec{v}_s$ is the space vector stator voltage. The parameters required by this model are the stator resistance $R_s$, the stator inductance $L_s$, the rotor resistance $R_r$, the rotor inductance $L_r$, the coupling stator-rotor inductance $L_{sr}$, the pole pairs $n_p$ and the moment of inertia $J$. $T_m$ is the mechanical load torque and $\omega_m$ is the mechanical speed. The coordinates transformation $\alpha\beta$ used in this paper is,

$$\vec{x} = x_\alpha + jx_\beta = \frac{2}{3} \left(x_a + e^{\frac{2\pi}{3}} x_b + e^{\frac{4\pi}{3}} x_c\right)$$  \hspace{1cm} (4)

III. PARAMETERS ESTIMATION

A. Stator input impedance

The proposed method for estimating the equivalent circuit parameters of the induction machine is based on the instantaneous stator input impedance during the start-up. The machine model is expressed as function of the stator and rotor currents, nevertheless the rotor currents is not a measurable variable in the squirrel cage induction motor. The rotor current expressed as function of the stator leakages flux $\vec{\lambda}_s$ is,

$$\vec{\lambda}_s = L_s \vec{i}_s + L_{sr} \vec{i}_r$$  \hspace{1cm} (5)

$$\vec{i}_r = \frac{\vec{\lambda}_s}{L_s} - \frac{L_{sr}}{L_s} \vec{i}_s$$  \hspace{1cm} (6)

Replacing (6) in (1) and (2), the following expressions are obtained,

$$p\vec{\lambda}_s = \vec{v}_s - R_s \vec{i}_s$$  \hspace{1cm} (7)

$$0 = \left(\frac{R_s}{L_s} - jn_p \omega_m\right)\vec{\lambda}_s + p\vec{\lambda}_s + \cdots$$

$$\cdots - \left(\frac{R_s}{L_r} - jn_p \omega_m\right)\vec{i}_s - L_s p\vec{i}_s$$  \hspace{1cm} (8)

where, $\vec{L}_s = L_s - \vec{L}_s / L_s$. Finally, the instantaneous stator input impedance is found from (7) and (8) as,

$$z_{in} = \frac{\vec{v}_s}{\vec{i}_s} = R_s + R_s \frac{L_s}{L_r} - jn_p \omega_m \vec{L}_s + \vec{L}_s \vec{p}\vec{i}_s + \cdots$$

$$\cdots - \left(\frac{R_s}{L_r} - jn_p \omega_m\right)\vec{i}_s - L_s p\vec{i}_s$$  \hspace{1cm} (9)

The instantaneous stator input impedance of the induction machine $z_{in}$ depends on the stator space vector voltage, current and flux leakage, as well as the machine parameters. The instantaneous stator voltage and current are measured directly in the machine terminals. Moreover, the stator flux leakage is determined by integrating (7),

$$\vec{\lambda}_s (t) = \int_0^t \left(\vec{v}_s (\tau) - R_s \vec{i}_s (\tau)\right) d\tau$$  \hspace{1cm} (10)

B. Sensorless speed estimation

During the motor start-up the acceleration depends on the accelerating torque, which is defined in (3) as the difference between the electrical $T_e$ and load torque. It is possible to estimate the electrical torque using the stator current and the stator flux leakage as follows,

$$T_e = n_p \left(\vec{\lambda}_s \times \vec{i}_s\right)$$  \hspace{1cm} (11)

Also, if the start-up is without mechanical load, only the friction losses are acting as the mechanical motor load and can be modeled as a linear function of the angular speed $\omega_m$.

$$T_m = k\omega_m$$  \hspace{1cm} (12)

$$T_e - T_m = J_p \omega_m \Rightarrow T_e = J_p \omega_m + k\omega_m$$  \hspace{1cm} (13)

The angular speed is estimated by integrating the accelerating torque ($T_e - T_m$), during the motor start-up. The accelerating torque for the no load condition corresponds to the electric torque, shown in Fig. 1a. When the start up process reaches steady state operation, the motor has a constant angular speed $\omega_s$, because the net torque is zero, the electrical torque and losses torque are equal. This condition can be used to determine the accelerating torque. For any time $t \geq t_0$, the electric torque integration results in a ramp function in time, with constant slope $k\omega_s$, and the bias is the synchronous angular speed $J\omega_s$, as is observed in Fig. 1b. As the machine starts-up in the no load condition, the steady state angular speed $\omega_m$ is considered very close to synchronous speed $\omega_m = \omega_s$.

Fig. 1c shows the estimation of the angular speed for a three phase 200 hp, 460 V, 2 pole pair induction motor. The axis in Fig. 1 are expressed in per unit of the rated values. The machine’s inertia was estimated as $J_{est} = 2.5773 \text{kgm}^2$ with a 0.83% error. The coefficient $k_{est} = 0.04789 \text{Nms/rad}$ was estimated with a $5.6 \times 10^{-6}$% error.
C. Cost function $\Psi$

The cost function $\Psi$, used for the parameter estimation, is obtained with the square of the difference between the measured instantaneous input impedance at the motor terminals and the calculated one using the machine model.

$$\Psi = \sum_{k=1}^{n} \left( \frac{z_{med_k} - z_{cal_k}}{z_{med_k}} \right) \left( \frac{z_{med_k} - z_{cal_k}}{z_{med_k}} \right)^*$$  \hspace{1cm} (14)

where,

$$z_{med_k} = \frac{\bar{v}_{s_k}}{\bar{i}_{s_k}}$$  \hspace{1cm} (15)

and, the instantaneous input impedance $z_{cal_k}$ is obtained using (9).

During the motor start-up there is enough information to estimate many of the machine parameters. The input impedance changes during this process, this impedance has a small value when the machine is accelerating, and increases its value when the steady state is reached. In the transitional period, the effect of the leakage inductance and the rotor resistance is predominant and the electric torque expression is mainly determined by the rotor resistance value. However, in the steady state the magnetizing inductance has the main effect over the leakage inductance, because the current used to establish the flow can reach 50% of the rated current, especially if the start-up is at no load or with a reduced mechanical load torque.

Two measured line to line voltages and line currents are required to obtain the stator space vectors $\vec{v}_s$ and $\vec{i}_s$.

$$\vec{v}_s = \sqrt{\frac{2}{3}} \left( v_{ab} + e^{j\frac{2\pi}{3}} v_{bc} + e^{j\frac{4\pi}{3}} v_{ca} \right)$$  \hspace{1cm} (16)

$$\vec{i}_s = \sqrt{\frac{2}{3}} \left( i_a + e^{j\frac{2\pi}{3}} i_b + e^{j\frac{4\pi}{3}} i_c \right)$$  \hspace{1cm} (17)

where, $v_{ab} + v_{bc} + v_{ca} = 0$ and $i_a + i_b + i_c = 0$.

From the instantaneous stator line voltage and current during the motor start-up, using a sampling acquisition rate set at 18 kHz, a non-linear optimization was accomplished with $\Psi$ as the objective function. Table I shows the results for two methods. In the first method there are no previous assumptions about the parameters, that is all the model parameters are unknown. While in the second method $R_s$ is assumed to be known, because this parameter is always measurable. The angular speed is estimated as explained in the previous section.

The results show that the proposed methodology allows estimate the machine parameters accurately. In the first method the error is less than 10%, and the second method the error is less than 1%. These results confirm that is possible to estimate the parameters from the instantaneous stator impedance during the motor start-up. The classic parameter estimation method using the no load and rotor block test are difficult to perform in field. The main advantage of the proposed method is the use of instantaneous voltages and currents measured during a regular motor start-up.
The analyzed angular speed estimation method proposed in this work, uses the instantaneous voltage and currents measured during the induction motor start-up, to obtain accurate space vector model parameters, including the motor inertia and the number of iterations required to obtain a convergent solution. The method requires the numerical evaluation of the stator space vector current derivative $p_{1s}$, and the angular speed estimation by the integration of the instantaneous electrical torque $T_e$. The method is a very powerful tool for the off line parameter estimation because presents a considerable computational burden.

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**REFERENCES**


**TABLE I: Parameter estimation result**

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<th>Parameter</th>
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<th>%Error</th>
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</table>

**Figure 2: Instantaneous measured impedance $z_{medk}$ and the error between $z_{calc}$ during the start-up**

Fig. 2 shows the measured instantaneous impedance and the instantaneous error between the measured and calculated impedance after the estimation of the machine’s parameters.

**IV. CONCLUSION**

The analyzed angular speed estimation method proposed in this work, uses the instantaneous voltage and currents measured during the induction motor start-up, to obtain accurate space vector model parameters, including the motor inertia estimation and friction losses coefficient. Direct measurement of the stator resistance $R_s$ reduces the error function cost $\Psi$ and the number of iterations required to obtain a convergent solution.