FAST DEFUZZIFICATION METHOD BASED ON CENTROID ESTIMATION

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ABSTRACT

A new fast defuzzification method is presented. It is based on estimating the centroid position by fitting the fuzzy output shape into one single triangle. In regard to the computational time, this method is comparable to the "Bisector" method and provides better dynamical behavior. This new method was tested to control a second order plant and a benchmark Continuous Stirred Tank Reactor (CSTR).

KEYWORDS

Defuzzification methods, Centroid estimation.

1. INTRODUCTION

Fuzzy inference systems have been widely applied to process control [1]. In several applications, the dynamics of the system are fast enough to make difficult a good control performance when using typical computational equipment. Special attention should be given to defuzzification process when available computational capabilities are restricted by equipment size or cost. In these cases, the computational time must be reduced in order to improve the controller performance. Hence, it is important to use fast defuzzification methods [2]. One of the most accepted defuzzification methods, when using Mamdani fuzzy inference systems, is based on computing the centroid of the output area (see equation 1).

$$\overline{x} = \frac{\sum x \cdot f(x)}{\sum f(x)} \quad (1)$$

However, the relatively high computational requirements could be a limitation for several applications. As an alternative, faster and simple methods can be used such as finding the "mean of maxima" (MOM) or by finding the half-area point ("Bisector") [3].

This article presents a simple fast method for computing a "**C**entroid **A**pproximation" by fitting the fuzzy output area into a "**T**riangular" shape (CAT), see figure 1.

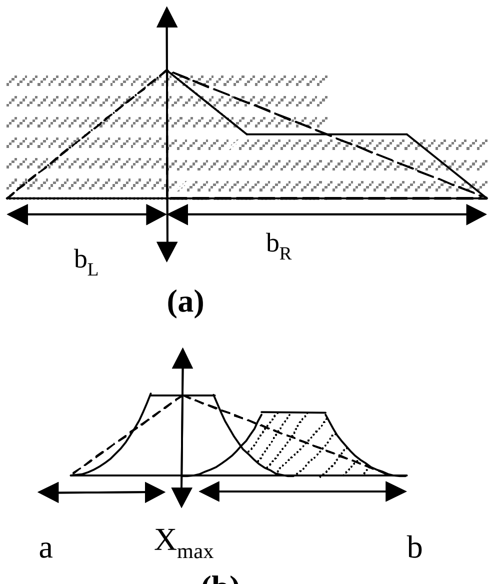

Fig. 1 Output fitting to triangular shapes.

2.METHOD DESCRIPTION

Wang and Luoh demonstrated [2] that for any triangular shape the centroid position could be computed as in equation 2.

$$\overline{x_t} = \frac{1}{3}\left(a + x_{\max} + b\right) \qquad (2)$$

They introduced a method that reduces the computational time by decomposing the output in triangles and trapezoids, and then computing and combining its centroids. This approach is a good solution, but presents two limitations. First, the output area has to be decomposed only in triangular and trapezoidal shapes, which cannot be possible when the membership functions do not have a triangular form, as an example the output in figure 1(b). Second, this technique needs to spend a computational time determining if each shape corresponds to a triangle or to a trapezoid. In this work a heuristic approach, which can even be applied for a wider set of memberships functions, is proposed. This approach consists in adapting **any output shape** into one single triangle. The computational time required by this algorithm is reduced with respect to that of the "Bisector". This approximation gives the exact centroid position for any cluster shapes having a base length and areas ratio of 1 to 3. For fuzzy outputs not located at the origin, the triangular shape maximum position is located at the maximum output shape position. To emphasize the triangular approximation, the equation 2 can be rewritten as equation 3.

$$\overline{x_t} = x_{\max} + \frac{1}{3}\left(b_R - b_L\right) \qquad (3)$$

When the fuzzy output presents more than one maximum, the location of the triangle maximum is computed as the average of maxima, as shown in figure 2(b).

2.1 ALGORITHM BASES

a) Find the first non-zero value, the last nonzero value and the maxima. This can be done in one single computation loop.

b) In case of several maxima find the average.

c) Determine b_R , b_L .

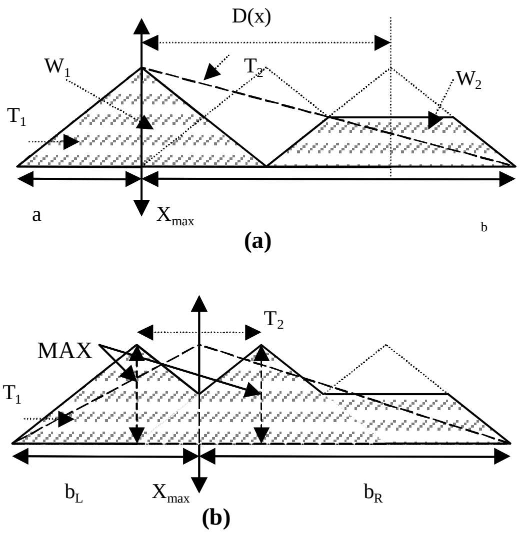

Fig. 2. (a) Two cluster fitting (b) Averaging two maxima

3. COMPUTED RESULTS

A simple analysis shows that the "MOM" method is very fast but produces high error for non-symmetrical outputs. The "Bisector" method is fast and generally produces good results but is inaccurate for asymmetrical shapes. In addition, this method requires two calculation sequences. The first determines the total area of the figure and the second determines the point corresponding to the half area. Unlike this, the proposed CAT method presents considerable advantages when the output is highly asymmetrical and especially for highly asymmetrical base lengths (see figure 2). More, the output calculation can be done in one single sequence.

Table 1
Comparison table for several defuzzification methods

	Cen	CAT	%Error	Bis	%Error	MM	%Error
Shape A	0	0	0	0	0	0	0
Shape B	0.19	0.17	1.04%	0.15	2.04%	0	9.38%
Shape C	0.43	0.33	4.76%	0.25	8.93%	0	21.43%

Table 1 shows the output for a symmetrical shape **A** centered at the origin and the shapes **B** and **C** correspond to Figures 1(a) and 2(a) respectively**.** The error shown in table 1 provided a favorable result for the proposed CAT method.

4. SIMULATION RESULTS

In order to show the method performance, several simulations were carried out using **Simulink®** in order to control a SISO linear second-order stable plant and a benchmark Continuous Stirred Tank Reactor (**CSTR**).

4.1 SECOND ORDER PLANT

Simulink® [3] brings already programmed in its fuzzy module the following methods: "Centroid", "Bisector" and "MOM". The **CAT** method was coded. Five triangular hard partitions were selected and the well-known control rule table president by [4] was used (see table 2). The control loop is shown in figure 3. The membership function for E and ΔE is shown in figure 4. For this problem a Set-point tracking was simulated. The **CAT** output was very similar to the "Centroid" output (see figure 5) and presented better dynamic response than the "Bisector".

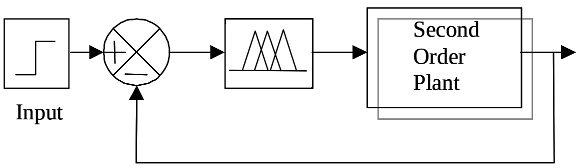

Fig. 3. Block diagram for the simulated plant

Table 2
Mac Vicar-Whelan table.

		E				
		ENB	ENS	EZ	EPS	EPB
ΔE	ΔENB	ΔUNB	ΔUNB	ΔUNM	ΔUNS	ΔUZ
	ΔENS	ΔUNB	ΔUNM	ΔUNS	ΔUZ	ΔUPS
	ΔEZ	ΔUNM	ΔUNS	ΔUZ	ΔUPS	ΔUPM
	ΔEPS	ΔUNS	ΔUZ	ΔUPS	ΔUPM	ΔUPB
	ΔEPB	ΔUZ	ΔUPS	ΔUPM	ΔUPB	ΔUPB

NB = Negative Big PS = Positive Small
NM = Negative Medium PM = Positive Medium
NS = Negative Small PB = Positive Big
Z = Zero

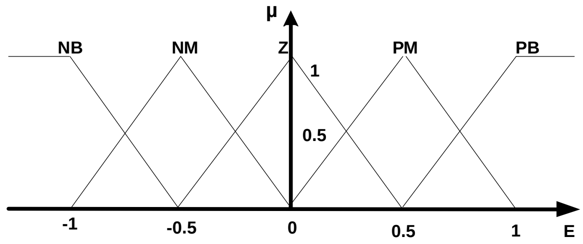

Fig. 4. Membership function

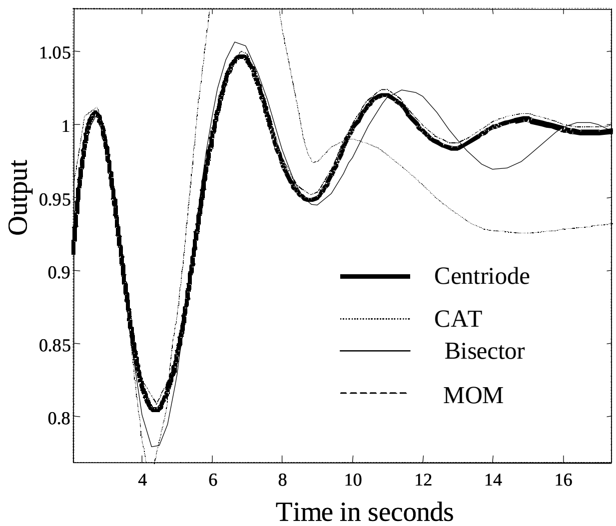

Fig. 5. Plant dynamical output

The output responses for the "Centroid" and "CAT" are so close that it is difficult to differentiate them from the plot in figure 5. The running time for the Centroid was 23 seconds and for the proposed method took only 0.99 seconds. The running time for the for "CAT" and "Bisector" are the same, but CAT yielded a better dynamical response getting at 5 % of the final value in 5.75 seconds, that is 3.75 seconds less than the "Bisector" method (see table 3).

Table 3

Running time and dynamical behavior for defuzzification methods

Method	**Running time**	**Dynamical Response**
Centroid	23.00 seconds	5.75 seconds
CETF	0.99 seconds	5.75 seconds
Bisector	0.99 seconds	9.50 seconds
M0M	1.00 seconds	∞

4.2 CONTINUOUS STIRRED TANK REACTOR

To verify the viability of the CAT method when applied to non-linear processes, it was tested to control a benchmark CSTR [5]. The description of the system is shown below. This process, shown in figure 6, is multivariable, highly coupled and nonlinear. Within the tank, the stir is supposed homogeneous, volume constant and to have the same characteristics at each point. These assumptions are stated following [5]. The exothermic reaction **A → B** takes place at the temperature value **T** and concentration values **Ca** and **Cb**. The molar input flow of the product **A**, labeled **F** has a concentration **Ca0** and its temperature is **T0**. The heat **Qk** is removed from the refrigerating flow by means of a cooling jacket. In this conditions energy and matter balance may be set for the whole reactor volume. The nonlinear equations that model the **CSTR** behavior are given below.

$$dCa/dt = F/V * (Ca0 - Ca) - k1_{(T)}Ca - k3_{(T)}Ca^2 \qquad (4)$$

$$dCb/dt = -F/V * Cb + k1_{(T)}Ca - k2_{(T)}Cb \qquad (5)$$

$$dT/dt = F/V (T0 - T) - (k1_{(T)}Ca\Delta H_{AB} + k2_{(T)}Cb\Delta H_{BC} + k3_{(T)}Ca^2\Delta H_{AD}) / \delta Cp - k_wAj/\delta CpV (T - Tj) \qquad (6)$$

$$dTj/dt = 1/mjCpj\,(Qk + k_wAj(T - Tj)) \qquad (7)$$

$$ki_{(T)} = k_{i0}\exp(Ei/T(^oC) + 273.15) \quad i = 1, 2, 3 \qquad (8)$$

The controlled process has two inputs ($F = u_1$ and $Qk = u_2$) and two outputs ($T = y_1$ and $Cb = y_2$). The proposed controller, whose architecture is shown in figure 7, consists mainly of two independent FIS (Fuzzy Inference System) and a static pre-compensator CM calculated using only the system open loop steady state gain matrix G [6].

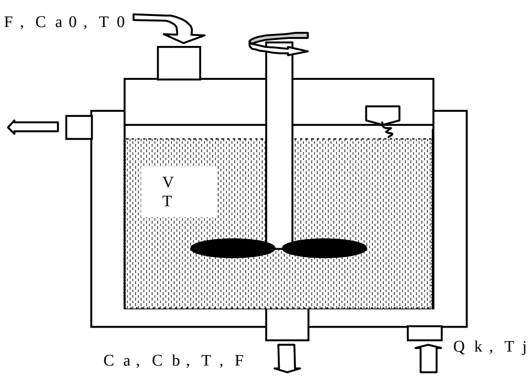

Fig. 6 CSTR process

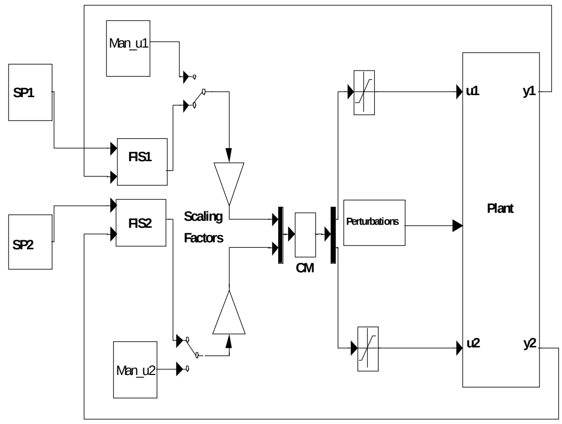

Fig.7 Controller Architecture

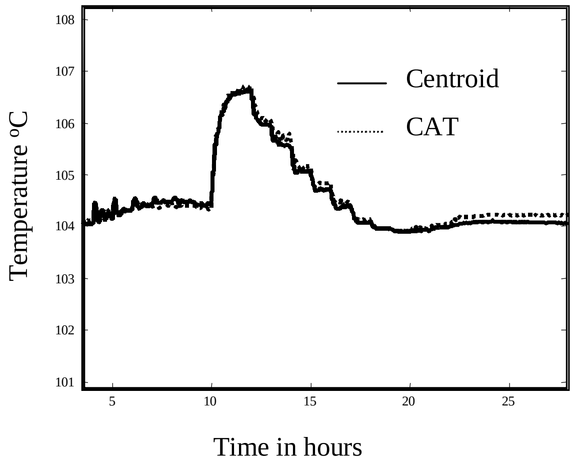

Fig 8. Temperature response (°C)

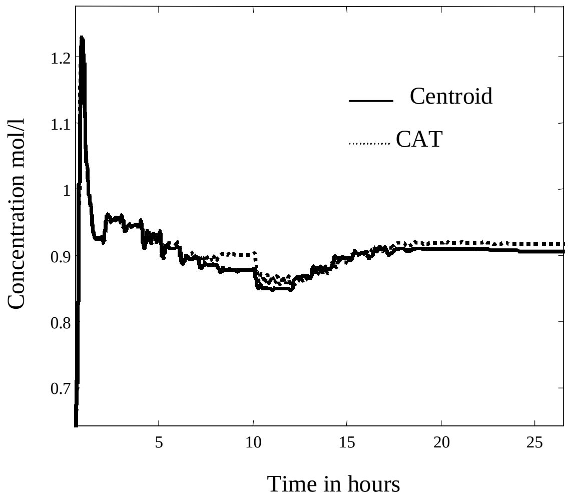

Fig. 9 Concentration response Cb (mol/l)

In this application, **FIS1** and **FIS2** are based on the same control rule table president by [4]. The control goal is to regulate the controlled variables **T** and **Cb** subject to some perturbations in **T0**. In figures 8 and 9 the plant closed-loop responses are shown when a perturbation in T0 occurs at 10 hours. The running time for the Centroid method was 7 minutes, 14 seconds and for the proposed method was only 37 seconds.

5. CONCLUSION

The proposed defuzzification method presents a good performance when applied to control a second order plant and a non-linear benchmark plant. It gave very short running time and a dynamical response very close to the one obtained with traditional "Centroid" defuzzification methods. Further work has to be done in order to optimize the algorithm performance and test the method with different types of membership functions.

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