A multivariable fuzzy controller for a Continuous Stirred Tank Reactor

SANCHEZ, Gustavo
Universidad Simón Bolívar, Departamento de Tecnología Industrial, Sartenejas, Edo. Miranda, Apartado Postal 89000, Venezuela. **E-mail**: gsanchez@usb.ve

CASTILLO, Eduardo
Universidad Nacional Experimental Politécnica de la Fuerza Armada Nacional (UNEFA), Venezuela.

STREFEZZA, Miguel
Universidad Simón Bolívar, Departamento de Procesos y Sistemas.

VILLEGAS, Thamara
Universidad Simón Bolívar, Departamento Electrónica y Circuitos.

REYES, Orlando
Universidad Central de Venezuela, Facultad de Ingeniería, Escuela de Ingeniería Mecánica.

ABSTRACT

In this work, a multivariable fuzzy controller for a Continuous Stirred Tank Reactor (**CSTR**) is presented. The interactive effects between inputs are partly handled by introducing a simple static linear decoupler. This technique simplifies the design procedure and overcomes the complexity potential in constructing the rule-base. A **Takagi-Sugeno** PI-like controller with parametric optimization of rules conclusions is proposed. Simulations, using a plant mathematical model, show good results taking to account the method simplicity.

KEY WORDS: Fuzzy Logic Control, Multi-Variable Control, Nonlinear Systems

1. INTRODUCTION

Linear SISO Process Fuzzy Controllers have been studied in depth. Many authors **[1]** have proposed tables that engineers may easily employ. They just need to set up the rules according to the experience of plant operators, if available. On the contrary, for **Nonlinear MIMO** Process, it becomes difficult to use conventional techniques (i.e. fully cross-coupling fuzzy controllers).

Perhaps the "*decentralized controller*" approach is the simplest that may be used to deal with this kind of problem. In 1995 J. Nie and D. Linkens **[2]** described a decentralized fuzzy controller structure for dealing with multivariable control of human blood pressure. In 1997, J. Nie **[3]** proposed a multivariable nonlinear controller for servomechanisms by means of fuzzy approaches. By using the concept of decentralized control, a structure is developed that is composed of two control loops, each of which is associated with a single-variable fuzzy controller and a decoupling unit. A simplified fuzzy control algorithm is used to implement the fuzzy controller. In 1999, P. Sarma **[4]**,**[5]** proposed a multivariable Gain-Scheduling fuzzy logic control of reactor processes. It is used with a simple effective static linear decoupler that permits simpler SISO Fuzzy Logic Controllers blocks to be used.

In our work a model free fuzzy controller is presented. A decentralized structure based on **[2]** is used in order to control a multivariable nonlinear **CSTR**. The interactive effects between inputs are partly handled by introducing a static compensation matrix calculated with the process experimental steady-state gain array.

In other hand, to improve designs of fuzzy controllers, many optimization methods have been proposed. In general form, these can be divided in two problems: First, structural optimization, which optimizes the number of rules and fuzzy sets to be used. Second, parametric optimization, which optimize the membership functions of the input and output variables, where these functions are supposed to be known. In this work, a parametric optimization technique, the *Stochastic Gradient Algorithm* proposed by P. Glorennec in **[6]**, is used in order to improve initial fuzzy rules.

This paper is divided in four parts. First, a mathematical description of the **CSTR** is given. Then, the controller design method is presented. Simulation results are provided in order to show the controller performance. Finally, some important remarks are given as conclusions.

2. PROCESS DESCRIPTION.

The controlled process consists on a Continuous Stirred Tank Reactor (**CSTR**). This process, shown in **Fig. 1**, is multivariable, highly coupled and nonlinear. Within the tank, the stir is supposed homogeneous, volume constant and to have the same characteristics in each point. These assumptions are stated following **[7]**. The exothermic reaction **A** ® **B** takes place at the temperature value **T** and concentration values **Ca** and **Cb**. The molar input flow of the product **A**, labeled **F** has a concentration **Ca0** and its temperature is **T0**. The heat **Qk** is removed from the refrigerating flow by means of a cooling jacket. In this conditions energy and matter balance may be set for the whole reactor volume. The nonlinear equations that model the **CSTR** behavior are given below.

$$dCa/dt = F/V*(Ca0 - Ca) - k1_{(T)}Ca - k3_{(T)}Ca^2 \qquad \textbf{(1)}$$

$$dCb/dt = -F/V*Cb + k1_{(T)}Ca - k2_{(T)}Cb \qquad \textbf{(2)}$$

$$dT/dt = F/V(T0 - T) - (k1_{(T)}Ca\Delta H_{AB} + k2_{(T)}Cb\Delta H_{BC} + k3_{(T)}Ca^2\Delta H_{AD})/dCp - k_wAj/dCpV(T - Tj) \qquad \textbf{(3)}$$

$$dTj/dt = 1/mjCpj(Qk + k_wAj(T - Tj)) \qquad \textbf{(4)}$$

$$ki_{(T)} = k_{i0}\exp(Ei/T(^oC) + 273.15) \quad i = 1, 2, 3 \qquad \textbf{(5)}$$

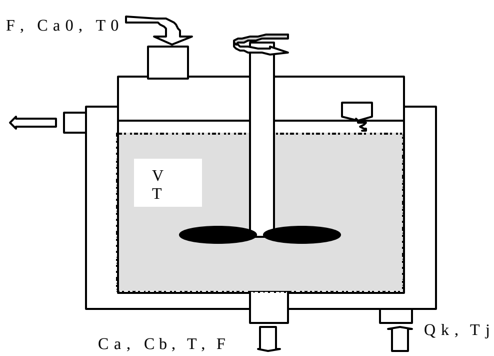


Fig. 1 CSTR process

Table 1. Parameters of the Model

Description	Symb	Value
Reaction Factor k_1	k_{10}	$(1.287 \pm 0.04)10^{12}\ h^{-1}$
Reaction Factor k_2	k_{20}	$(1.287 \pm 0.04)10^{12}\ h^{-1}$
Reaction Factor k_3	k_{30}	$(9.043 \pm 0.27)10^{9}\ h^{-1}l/molA$
Activation Energy k_1	E_1	$-9758.3\ K$
Activation Energy k_2	E_2	$-9758.3\ K$
Activation Energy k_3	E_3	$-8560\ K$
Reaction Entalpy k_1	ΔH_{AB}	$(4.2 \pm 2.36)\ kj/molA$
Reaction Entalpy k_2	ΔH_{BC}	$-(11.0 \pm 1.92)\ kj/molB$
Reaction Entalpy k_3	ΔH_{AD}	$-(41.84 \pm 1.41)\ kj/molA$
Density	d	$(0.9342 \pm 4\ 10^{-4})\ kg/l$
Heat capacity	Cp	$(3.01 \pm 0.04)\ kj/kg\ K$
Coolant Heat Transfer Coefficient	k_w	$(4032 \pm 120)\ kj\ h^{-1}/m^2K$
Coolling Jacket Surface	Aj	$0.215\ m^2$
Tank Volume	V	$0.01\ m^3$
Coolant Mass	m_j	$5.0\ kg$
Coolant Heat Capacity	Cpj	$(2.0 \pm 0.05)\ kj/kg\ K$

The parameter values employed for simulations are given in **Table 1**.

3.- CONTROLLER DESIGN METHOD.

A design method outline is shown in **Fig. 2**. The controlled process has two inputs (**F** and **Qk**) and two outputs (**Cb** and **T**). The proposed controller, whose architecture is shown in **Fig. 3**, consists mainly of two-separated **FIS** (Fuzzy Inference System) and a static pre-compensator **CM** calculated using only the system open loop steady state gain matrix **G**.

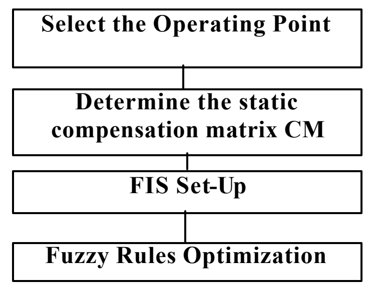


Fig. 2 Design Method Outline

The operating point must be chosen so that process behavior is approximately linear, or at least monotonic, near to the point. The operating point selected for this work is given in **Table 2.**

Table 2. Selected Operating Point

Ca0(mol/l)	**5.09**
T0(ºC)	**104.9**
T(ºC)	**104.2**
F	**30**
Cb(mol/l)	**0.91**
Qk(kJ/h)	**-1250**

In this application, **FIS1** and **FIS2** are **Takagi-Sugeno**, PI-like, and Zero Order type. For both **FIS**, the initial look-up table (**Table 3**) for the incremental output **Dui, i Î {1,2}** is taken from **[1]**.

Table 3. Initial Look-up Table.

Ei / ΔEi	NB	NS	Z	PS	PB
NB	-1	-1	-0.66	-0.33	0
NS	-1	-0.66	-0.33	0	0.33
Z	-0.66	-0.33	0	0.33	0.66
PS	-0.33	0	0.33	0.66	1
PB	0	0.33	0.66	1	1

P: Positive, N: Negative, S: Small, B: Big, Z: Zero

The control goal is to maintain the controlled variable $\mathbf{y_i}$, $\mathbf{i} \in \{1,2\}$, at the Set Point $\mathbf{SP_i}$, subject to some step perturbations. Let $\mathbf{\hat{u}_i}$ be the input value at the Operating Point. Then, the system steady state gain matrix **G** and the relative gain array **RGA** are calculated following equations **6** and **7**.

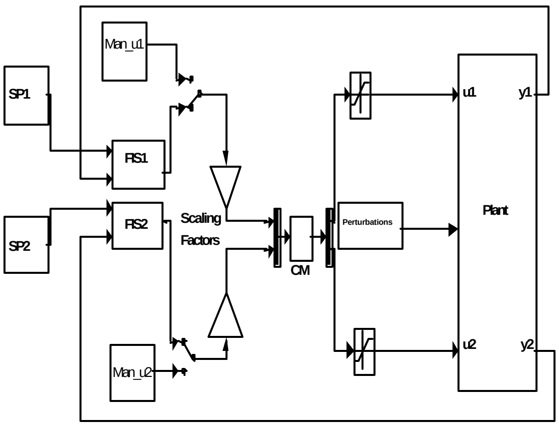


Fig.3 Controller Architecture

$$G = \begin{pmatrix} g11 & g12 \\ g21 & g22 \end{pmatrix} \ \mathbf{n = 2}\ ;\ gij = \left[\frac{\Delta yi}{\Delta uj}\right] \text{, with } \mathbf{u_k},\ k \neq j \text{, constant} \qquad \textbf{(6)}$$

$$RGA = \begin{pmatrix} \frac{g11g22}{\Delta} & -\frac{g12g21}{\Delta} \\ -\frac{g12g21}{\Delta} & \frac{g11g22}{\Delta} \end{pmatrix} \qquad \Delta = \det(G) \neq 0 \qquad \textbf{(7)}$$

The compensation matrix CM and the input value $\mathbf{\hat{u}_i}$* of the compensated system, are calculated following equations **8** and **9**.

$$CM = \begin{pmatrix} 1 & -\frac{g12}{g11} \\ -\frac{g21}{g22} & 1 \end{pmatrix},\ \det(CM) \neq 0 \qquad \textbf{(8)}$$

$$\begin{pmatrix} \hat{u}1^* \\ \hat{u}2^* \end{pmatrix} = CM^{-1} \begin{pmatrix} \hat{u}1 \\ \hat{u}2 \end{pmatrix} \qquad \textbf{(9)}$$

These equations allow establishing input influences on each output and selecting control "pairs" **($\mathbf{u_i}$, $\mathbf{y_i}$) i = 1,2**. This is done following the criterion presented in **[2]**, this is, to pair those variables whose relative gain is positive and as close to unity as possible.

The "error" $\mathbf{E_i}$ and "error change" $\mathbf{DE_i}$ are normalized, by means of scaling factors, between –1 and 1, taking to account operating restrictions. Membership functions of E_i and $\mathbf{D}E_i$ are shown in **Fig. 4**.

```
 NB      NS  |Z   PS      PB
____      \   /\   /|\   /\   /____
            \ / \ /|\ / \ /
             \/   \/ | \/   \/
             /\   /\ | /\   /\
            / \ / \|/ \ / \
           /   \/   \|/   \/   \
_____|___|___|___|__|____ Ei, DEi
   -1 -0.5  0 0.5  1
```

Fig.4 Normalized Membership Functions for $\mathbf{E_i}$ and $\mathbf{DE_i}$

Scaling factors are tuned heuristically. To optimize fuzzy rules, the *Stochastic Gradient Algorithm* is implemented **[5]**. It can be applied either to antecedents or to conclusions of the fuzzy rules. In this case, it is applied only for the conclusions. The equation that defines the optimization of the *i-th* conclusion is:

$$\Delta b_i = -\eta * E * \frac{\alpha_i}{\sum_j \alpha_j} \qquad \textbf{(10)}$$

where, $\mathbf{\Delta b_i}$ is the increment of the *i-th* conclusion; $\boldsymbol{\eta} \in [0,1]$ is the optimization step; **E** is the actual error; $\boldsymbol{\alpha}_i$ is the antecedent value of the *i-th* rule. This equation is calculated after each rule output is obtained, only if **E > Emax** (threshold user-selected value), to avoid undesirable oscillations in the process outputs. The optimization of each **FIS** is done independently.

4. - SIMULATION RESULTS.

The process equations **1-5** were implemented in SIMULINK® and a simulation was carried off. First, to determine the compensation matrix **CM**, a step was simulated for each input. The response of the plant is shown in **Fig. 5.a – 5.b** and **Fig. 6.a – 6.b**. The obtained steady state gains are shown in **Table 4**.

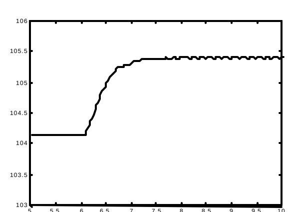


Fig 5.a

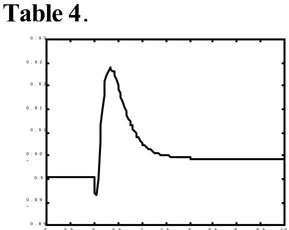


Fig 5.b

Response of the plant when a +10% step in the input **F** occurs at 6 hours.(a) T (ºC) ; (b) Cb (mol/ l)

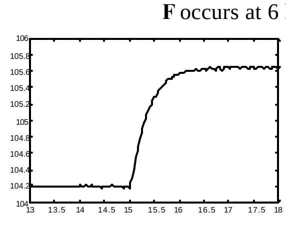


Fig 6.a

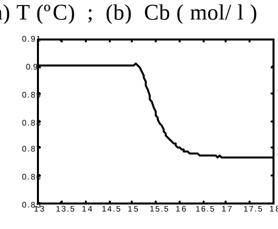


Fig 6.b

Response of the plant when a +10% step in the input **Qk** occurs at 15 hours.(a) T (ºC) ; (b) Cb (mol/ l)

Table 4. **Steady state gains**

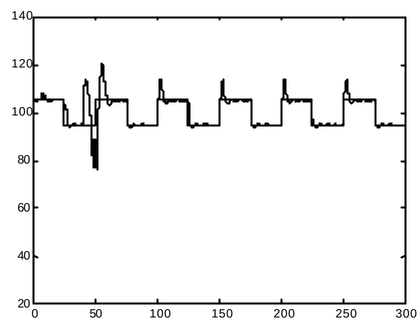
	ΔInput	ΔOutput	ΔOutput/ΔInput
F/Cb	3	0.0044	0.0015
F/T	3	1.26	0.42
Qk/Cb	125	-0.037	-2.96e-004
Qk/T	125	1.08	0.0086

Table 4 allows determining numerical values for matrix **G**, **RGA** and **CM**.

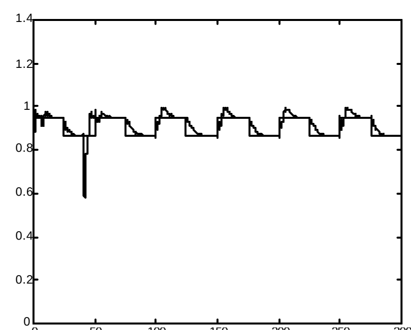
$$G = \begin{pmatrix} 0.42 & 0.0086 \\ 0.0015 & -2.96e-4 \end{pmatrix} \quad RGA = \begin{pmatrix} 0.906 & 0.094 \\ 0.094 & 0.906 \end{pmatrix}$$

$$CM = \begin{pmatrix} 1 & \text{-0.0205} \\ 5.0676 & 1 \end{pmatrix} \quad \begin{pmatrix} F \\ Qk \end{pmatrix} = \begin{pmatrix} u1 \\ u2 \end{pmatrix} \quad \begin{pmatrix} T \\ Cb \end{pmatrix} = \begin{pmatrix} y1 \\ y2 \end{pmatrix}$$

The optimization of conclusions was implemented using a square wave signal (**Fig. 7.a , 7.b**) as reference first for the pair (F,T) with Emax = 0.1 and η = 0.01 and then for the pair (Qk,Cb), with Emax = 0.01 and η = 0.01.

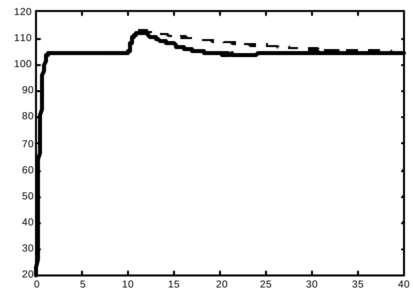


7.a
T(ºC)

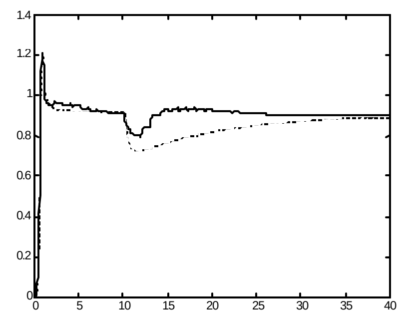


7.b
T(ºC)

Finally, the controller ability to reject some step perturbations was tested. In **Fig. 8.a** and **8.b** the controller performance, with and without rule conclusions optimization, is shown.



8.a



8.b

Fig.8
Response of the plant when a perturbation in T0 occurs at 10 hours
______ with optimization **.......** without optimization
(a) T(°C) ; (b) Cb(mol/l)

The 2-norm of error signals show that optimization improves the controller performance **(Table 5)**.

Table 5. Error 2-norm

	Without optimization	With optimization
$\|E_T\|_2$	**496.5**	**483.7**
$\|E_{Cb}\|_2$	**6.4**	**5.9**

5. CONCLUSIONS

The presented method shows good results and has a major advantage because of its simplicity. It is not based on an explicit system mathematical model. However, it can be applied only if the steady state matrix gain is nonsingular and also if it may be determined by means of experimental or theoretical approaches (for example, trials with the open loop system). The method could be easily implemented with standard industrial controllers. The introduced decoupling matrix does not eliminate the interaction completely, but reduces it to acceptable levels.

In this application, the chosen operating point is not an “optimum” one, like recommended in **[8]**. In fact, tests proved that our method works better in this point, because system behavior is more “linear. Rule-base optimization was carried off in a very simple way, but results are good enough.

Scaling factors tuning is very important to improve the fuzzy controller performance. Auto-tuning by means of Gain-Scheduling may be an excellent goal for further developments.

REFERENCES

[1] D. Driankov, *An introduction to Fuzzy-Control*. (Springer-Verlag. Berlin – Heidelberg, 1993).

[2] J. Nie and D. Linkens, *Fuzzy-Neural Control Principles, Algorithms and Applications. Chapter 3, pp. 37-67* (Prentice Hall International, 1995)

[3] J. Nie, Fuzzy control of multivariable nonlinear servomechanisms with explicit decoupling scheme , *IEEE TFS Vol. 5* No 2 pp. 304-11, May 1997

[4] P. Sarma, Multivariable Gain-Scheduled Fuzzy Logic Control of an Exothermic Reactor , *Engineering Applications of Artificial Intelligence,* accepted 1999.

[5] P. Sarma, R. Raghunathan, Multivariable Gain-Scheduled Fuzzy Logic Control of a Fluidized Catalytic Cracker Unit , *7th Symposium on Process Systems Engineering,* Keystone, PSE 2000, Colorado, USA.

[6] P. Glorennec, *Algorithmes d’ apprentissage pour systèmes d’ inférance floue.* (Hermes Science Publications, Paris. 1999)

[7] T. Durán, *Control Predictivo Tolerante a Fallos Reconfigurable en un Proceso No Lineal*. (Universidad de Valladolid, España. Dpto. de Ingeniería de Sistemas y Automática. Tesis presentada para optar al grado de Máster en Sistemas de Control Industrial. Junio de 2000)

[8] H. Chen, A. Kremling and F. Allgower, Non linear predictive control of a benchmark CSTR. *Proceeding of 3rd, European Control Conference,* Rome, Italy, Sept. 1995