A multivariable fuzzy controller for a Continuous Stirred Tank Reactor

SANCHEZ, Gustavo
Universidad Simón Bolívar, Departamento de Tecnología Industrial, Sartenejas, Edo. Miranda, Apartado Postal 89000, Venezuela. E-mail: gsanchez@usb.ve

CASTILLO, Eduardo
Universidad Nacional Experimental Politécnica de la Fuerza Armada Nacional (UNEFA), Venezuela.

STREFEZZA, Miguel
Universidad Simón Bolívar, Departamento de Procesos y Sistemas.

VILLEGAS, Thamara
Universidad Simón Bolívar, Departamento Electrónica y Circuitos.

REYES, Orlando
Universidad Central de Venezuela, Facultad de Ingeniería, Escuela de Ingeniería Mecánica.

ABSTRACT
In this work, a multivariable fuzzy controller for a Continuous Stirred Tank Reactor (CSTR) is presented. The interactive effects between inputs are partly handled by introducing a simple static linear decoupler. This technique simplifies the design procedure and overcomes the complexity potential in constructing the rule-base. A Takagi-Sugeno PI-like controller with parametric optimization of rules conclusions is proposed. Simulations, using a plant mathematical model, show good results taking to account the method simplicity.

KEY WORDS: Fuzzy Logic Control, Multi-Variable Control, Nonlinear Systems

1. INTRODUCTION

Linear SISO Process Fuzzy Controllers have been studied in depth. Many authors [1] have proposed tables that engineers may easily employ. They just need to set up the rules according to the experience of plant operators, if available. On the contrary, for Nonlinear MIMO Process, it becomes difficult to use conventional techniques (i.e. fully cross-coupling fuzzy controllers).

Perhaps the “decentralized controller” approach is the simplest that may be used to deal with this kind of problem. In 1995 J. Nie and D. Linkens [2] described a decentralized fuzzy controller structure for dealing with multivariable control of human blood pressure. In 1997, J. Nie [3] proposed a multivariable nonlinear controller for servomechanisms by means of fuzzy approaches. By using the concept of decentralized control, a structure is developed that is composed of two control loops, each of which is associated with a single-variable fuzzy controller and a decoupling unit. A simplified fuzzy control algorithm is used to implement the fuzzy controller. In 1999, P. Sarma [4],[5] proposed a multivariable Gain-Scheduling fuzzy logic control of reactor processes. It is used with a simple effective static linear decoupler that permits simpler SISO Fuzzy Logic Controllers blocks to be used.

In our work a model free fuzzy controller is presented. A decentralized structure based on [2] is used in order to control a multivariable nonlinear CSTR. The interactive effects between inputs are partly handled by introducing a static compensation matrix calculated with the process experimental steady-state gain array.

In other hand, to improve designs of fuzzy controllers, many optimization methods have been proposed. In general form, these can be divided in two problems: First, structural optimization, which optimizes the number of rules and fuzzy sets to be used. Second, parametric optimization, which optimize the membership functions of the input and output variables, where these functions are supposed to be known. In this work, a parametric optimization technique, the Stochastic Gradient Algorithm proposed by P. Glorennec in [6], is used in order to improve initial fuzzy rules.

This paper is divided in four parts. First, a mathematical description of the CSTR is given. Then, the controller design method is presented. Simulation results are provided in order to show the controller performance. Finally, some important remarks are given as conclusions.

2. PROCESS DESCRIPTION.

The controlled process consists on a Continuous Stirred Tank Reactor (CSTR). This process, shown in Fig. 1, is multivariable, highly coupled and nonlinear. Within the tank, the stir is supposed homogeneous, volume constant and to have the same characteristics in each point. These
assumptions are stated following [7]. The exothermic reaction \( \text{A} \rightarrow \text{B} \) takes place at the temperature value \( T \) and concentration values \( C_a \) and \( C_b \). The molar input flow of the product \( \text{A} \), labeled \( F \), has a concentration \( C_{a0} \) and its temperature is \( T_0 \). The heat \( Q_k \) is removed from the refrigerating flow by means of a cooling jacket. In this conditions energy and matter balance may be set for the whole reactor volume. The nonlinear equations that model the CSTR behavior are given below.

\[
\frac{dC_a}{dt} = F \cdot (C_{a0} - C_a) - k_1(T)C_a - k_3(T)C_a^2 \quad (1)
\]

\[
\frac{dC_b}{dt} = -F \cdot C_b + k_1(T)C_a - k_2(T)C_b \quad (2)
\]

\[
\frac{dT}{dt} = \frac{F}{V}(T_0 - T) - (k_1(T)C_a + k_2(T)C_b) + k_3(T)C_a^2(\Delta H_{AD}/\Delta C_p) + k_{wA}(T - T_j) \quad (3)
\]

\[
\frac{dC_b}{dt} = \frac{F}{V}(T_0 - T) - (k_1(T)C_a + k_2(T)C_b) + k_3(T)C_a^2(\Delta H_{AD}/\Delta C_p) + k_{wA}(T - T_j) \quad (4)
\]

\[
k_i(T) = k_{i0} \exp\left(\frac{E_i}{R(T_0)} + 273.15\right) \quad i = 1, 2, 3 \quad (5)
\]

The parameter values employed for simulations are given in Table 1.

### 3.- CONTROLLER DESIGN METHOD.

A design method outline is shown in Fig. 2. The controlled process has two inputs (\( F \) and \( Q_k \)) and two outputs (\( C_b \) and \( T \)). The proposed controller, whose architecture is shown in Fig. 3, consists mainly of two-separated FIS (Fuzzy Inference System) and a static pre-compensator CM calculated using only the system open loop steady state gain matrix \( G \).

- **Select the Operating Point**
- **Determine the static compensation matrix CM**
- **FIS Set-Up**
- **Fuzzy Rules Optimization**

![Fig. 2 Design Method Outline](image)

The operating point must be chosen so that process behavior is approximately linear, or at least monotonic, near to the point. The operating point selected for this work is given in Table 2.

<table>
<thead>
<tr>
<th>Description</th>
<th>Symb</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Re action Factor ( k_1 )</td>
<td>( \xi_1 )</td>
<td>(1.237 ± 0.04) ( \times 10^{-4} ) ( s^{-1} )</td>
</tr>
<tr>
<td>Re action Factor ( k_2 )</td>
<td>( \xi_2 )</td>
<td>(1.237 ± 0.04) ( \times 10^{-4} ) ( s^{-1} )</td>
</tr>
<tr>
<td>Activation Energy ( k_3 )</td>
<td>( \xi_3 )</td>
<td>(1.843 ± 0.25) ( \times 10^{-7} ) ( s^{-1} )</td>
</tr>
<tr>
<td>Activation Energy ( k_4 )</td>
<td>( \xi_4 )</td>
<td>(1.843 ± 0.25) ( \times 10^{-7} ) ( s^{-1} )</td>
</tr>
<tr>
<td>Reaction Entalpy ( k_5 )</td>
<td>( \xi_5 )</td>
<td>(3.519 ± 0.25) ( \times 10^{-5} ) ( s^{-1} )</td>
</tr>
<tr>
<td>Reaction Entalpy ( k_6 )</td>
<td>( \xi_6 )</td>
<td>(3.519 ± 0.25) ( \times 10^{-5} ) ( s^{-1} )</td>
</tr>
<tr>
<td>Density</td>
<td>( \xi_7 )</td>
<td>(1.500 ± 0.05) ( \times 10^{-7} ) ( kg^{-1} )</td>
</tr>
<tr>
<td>Coolant Heat Transfer Coefficient ( c )</td>
<td>( \xi_8 )</td>
<td>(1.500 ± 0.05) ( \times 10^{-7} ) ( kg^{-1} )</td>
</tr>
<tr>
<td>Cooling Jacket Surface ( F )</td>
<td>( \xi_9 )</td>
<td>(1.500 ± 0.05) ( \times 10^{-7} ) ( kg^{-1} )</td>
</tr>
<tr>
<td>Task Volume</td>
<td>( \xi_{10} )</td>
<td>(1.500 ± 0.05) ( \times 10^{-7} ) ( kg^{-1} )</td>
</tr>
<tr>
<td>Coolant Mass</td>
<td>( \xi_{11} )</td>
<td>(1.500 ± 0.05) ( \times 10^{-7} ) ( kg^{-1} )</td>
</tr>
</tbody>
</table>

In this application, FIS1 and FIS2 are Takagi-Sugeno, PI-like, and Zero Order type. For both FIS, the initial look-up table (Table 3) for the incremental output \( \Delta u_i \), \( i \in \{1, 2\} \) is taken from [1].

<table>
<thead>
<tr>
<th>( E_i )</th>
<th>NB</th>
<th>NS</th>
<th>Z</th>
<th>PS</th>
<th>PB</th>
</tr>
</thead>
<tbody>
<tr>
<td>NB</td>
<td>-1</td>
<td>-1</td>
<td>-0.66</td>
<td>-0.33</td>
<td>0</td>
</tr>
<tr>
<td>NS</td>
<td>-1</td>
<td>-0.66</td>
<td>-0.33</td>
<td>0</td>
<td>0.33</td>
</tr>
<tr>
<td>Z</td>
<td>-0.66</td>
<td>-0.33</td>
<td>0</td>
<td>0.33</td>
<td>0.66</td>
</tr>
<tr>
<td>PS</td>
<td>-0.33</td>
<td>0</td>
<td>0.33</td>
<td>0.66</td>
<td>1</td>
</tr>
<tr>
<td>PB</td>
<td>0</td>
<td>0.33</td>
<td>0.66</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

P: Positive, N: Negative, S: Small, B: Big, Z: Zero
The control goal is to maintain the controlled variable \( y_i, i \in \{1,2\} \), at the Set Point \( S_P_i \), subject to some step perturbations. Let \( \hat{u}_i \) be the input value at the Operating Point. Then, the system steady state gain matrix \( G \) and the relative gain array \( RGA \) are calculated following equations 6 and 7.

\[
G = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix}, n = 2 ; g_{ij} = \frac{\Delta y_i}{\Delta u_j}, \text{with } u_k, k \neq j, \text{ constant (6)}
\]

\[
RGA = \begin{pmatrix} \frac{g_{11}g_{22} - g_{12}g_{21}}{\Delta} & -\frac{g_{12}g_{21}}{\Delta} \\ -\frac{g_{12}g_{21}}{\Delta} & \frac{g_{11}g_{22} - g_{12}g_{21}}{\Delta} \end{pmatrix}, \Delta = \det(G) \neq 0 \text{ (7)}
\]

The compensation matrix \( CM \) and the input value \( \hat{u}^*_i \) of the compensated system, are calculated following equations 8 and 9.

\[
CM = \begin{pmatrix} 1 & g_{12} \\ -g_{21} & 1 \end{pmatrix} , \det(CM) \neq 0 \text{ (8)}
\]

\[
\begin{pmatrix} \hat{u}_1^* \\ \hat{u}_2^* \end{pmatrix} = CM^{-1} \begin{pmatrix} \hat{u}_1 \\ \hat{u}_2 \end{pmatrix} \text{ (9)}
\]

These equations allow establishing input influences on each output and selecting control “pairs” \( (u_i, y_j) i = 1,2 \). This is done following the criterion presented in [2], this is, to pair those variables whose relative gain is positive and as close to unity as possible.

The “error” \( E_i \) and “error change” \( \Delta E_i \) are normalized, by means of scaling factors, between –1 and 1, taking to account operating restrictions. Membership functions of \( E_i \) and \( \Delta E_i \) are shown in Fig. 4.

4. - SIMULATION RESULTS.

The process equations 1-5 were implemented in SIMULINK® and a simulation was carried off. First, to determine the compensation matrix \( CM \), a step was simulated for each input. The response of the plant is shown in Fig. 5.a – 5.b and Fig. 6.a – 6.b. The obtained steady state gains are shown in Table 4.
Table 4. Steady state gains

<table>
<thead>
<tr>
<th></th>
<th>ΔInput</th>
<th>ΔOutput</th>
<th>ΔOutput/ΔInput</th>
</tr>
</thead>
<tbody>
<tr>
<td>F/Cb</td>
<td>3</td>
<td>0.0044</td>
<td>0.0015</td>
</tr>
<tr>
<td>F/T</td>
<td>3</td>
<td>1.26</td>
<td>0.42</td>
</tr>
<tr>
<td>Qk/Cb</td>
<td>125</td>
<td>-0.037</td>
<td>-2.96e-04</td>
</tr>
<tr>
<td>Qk/T</td>
<td>125</td>
<td>1.08</td>
<td>0.0086</td>
</tr>
</tbody>
</table>

Table 4 allows determining numerical values for matrix G, RGA and CM.

\[
G = \begin{pmatrix} 0.42 & 0.0086 \\ 0.0015 & -2.96e-4 \end{pmatrix}, \quad RGA = \begin{pmatrix} 0.906 & 0.094 \\ 0.094 & 0.906 \end{pmatrix}
\]

\[
CM = \begin{pmatrix} 1 & -0.0205 \\ 5.0676 & 1 \end{pmatrix}, \quad \begin{pmatrix} F \\ Qk \end{pmatrix} = \begin{pmatrix} ut \\ u2 \end{pmatrix}, \quad \begin{pmatrix} T \\ Cb \end{pmatrix} = \begin{pmatrix} y1 \\ y2 \end{pmatrix}
\]

The optimization of conclusions was implemented using a square wave signal (Fig. 7.a , 7.b) as reference first for the pair (F,T) with \( E_{max} = 0.1 \) and \( \eta = 0.01 \) and then for the pair (Qk,Cb), with \( E_{max} = 0.01 \) and \( \eta = 0.01 \).

Finally, the controller ability to reject some step perturbations was tested. In Fig. 8.a and 8.b the controller performance, with and without rule conclusions optimization, is shown.

<table>
<thead>
<tr>
<th></th>
<th>Without optimization</th>
<th>With optimization</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_T |_2 )</td>
<td>496.5</td>
<td>483.7</td>
</tr>
<tr>
<td>( E_Cb |_2 )</td>
<td>6.4</td>
<td>5.9</td>
</tr>
</tbody>
</table>

5. CONCLUSIONS

The presented method shows good results and has a major advantage because of its simplicity. It is not based on an explicit system mathematical model. However, it can be applied only if the steady state matrix...
gain is nonsingular and also if it may be determined by means of experimental or theoretical approaches (for example, trials with the open loop system). The method could be easily implemented with standard industrial controllers. The introduced decoupling matrix does not eliminate the interaction completely, but reduces it to acceptable levels.

In this application, the chosen operating point is not an “optimum” one, like recommended in [8]. In fact, tests proved that our method works better in this point, because system behavior is more “linear. Rule-base optimization was carried off in a very simple way, but results are good enough.

Scaling factors tuning is very important to improve the fuzzy controller performance. Auto-tuning by means of Gain-Scheduling may be an excellent goal for further developments.

REFERENCES


